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Nuclear Deformation Shape of Even- Even $^{122-146}$Ba and $^{200-214}$Pb Isotopes

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A B S T R A C T

In this paper, the shape for even-even $^{122-146}\text{Ba}$ and $^{200-214}\text{Pb}$ nuclei have been studied, by using deformed shell model equations, to calculate the reduced electric transition probability $B(E2)\uparrow$,between the ground state and first excited state to calculate nucleus quadrupole deformation parameter β for understanding the shape of nuclei as well as the intrinsic quadrupole moment Q_0 and the root mean square of the radius $\langle r^2 \rangle^{1/2}$ were calculated, and have been used through calculating major (a) and minor (b)ellipsoid axes and the different between them ΔR . The relationship between the difference of the major and minor elliptical axes (a,b) and neutrons number was studied. It's clear that the ΔR and the deformation parameters β of $_{56}\text{Ba}$ and $_{82}\text{Pb}$ isotopes decreased when the neutrons number reach to the magic number $N = 82$ and $N = 126$. To achieve our mean purposes which determine the identify if the nucleus under the study has prolate, oblate or spherical shape, we have been compared our calculations with the stander values for the experimental ΔR_{exp} range. Finally, the nucleus under the study was plotted in two dimensions $2D$ depending on the value of (a,b) .

Keywords: Deformation Parameters; Difference of the major and minor elliptical axes; Electric Quadrupole Moment; Electric Transition Probability; Root Mean-Squared Radius.

1. Introduction

In the first half of the 20^{th} century many of models were introduced to explain the observed shells structure in nuclei. The nuclei which have magic numbers (2, 8, 20, 28, 50, 82 and 126) of nucleons protons and/or neutrons it will be more stable and has a spherical shape [1]. Where there are large energy gaps between successive nuclear orbitals, and the binding energy of the last nucleon is much larger than the corresponding value in the neighboring [2]. The deformation occurs only when both proton and neutron shells are partly filled [3]. In other word, the nucleus becomes deformed when the number of nucleons is not equal to the magic number. The nucleus shape can be determined by the intrinsic quadrupole moment Q_0, which is calculated only by measuring the reduced electric quadrupole transitions probability $B(E2)\uparrow$ between the ground and first excited states in even-even nuclides [4]. The intrinsic quadrupole moment Q_0 in even-even isotopes is takes three values, a small value that may reach zero for nuclei that have a spherically symmetric charge distribution, negative value for oblate nuclei and finally the positive value for prolate shape of the nuclei [5]. These shapes can be modified between neighboring nuclei by rearranging protons or neutrons within the same nucleus to change their shape. Therefore, the same nucleus can take different forms corresponding to different energy states [6]. The energy spectrum of spherical nuclei can be explained by the surface vibrations of the nucleus as shown in Figure (1). In this work, we calculated the deformation coefficient and the average radius of the charge distribution to find the major (a) and minor (b) elliptical axes and calculated the difference between them, and its effect on the shape of the nuclei (flat, swollen, or spherical) of barium and lead isotopes.

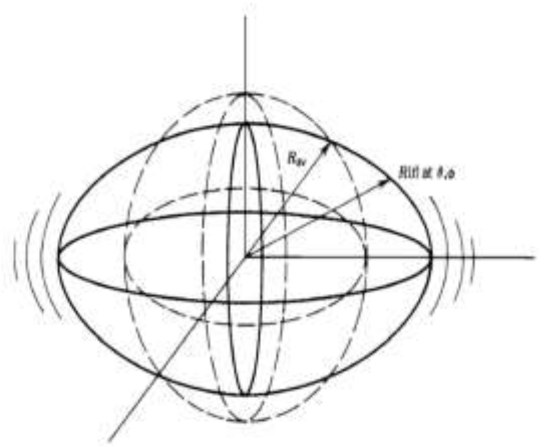

Fig.1. A circular equilibrium with a vibrating nucleus. A point on the surface in the (θ,ϕ) direction is located by the time- dependent coordinate R(t) [6]

1. Method of Calculation

Nucleus quadrupole deformation parameter β was obtained using the following equation [7-9]:

$$\beta = \frac{4\pi}{3ZR_0^2}\left[\frac{B(E_2)\uparrow}{e^2}\right]^{1/2} \qquad (1)$$

Where:

Z is the atomic number of the isotope and R_0 is average the nuclear charge radii which calculated by the following formula [10] :

$$R_0^2 = \left(1.2A^{1/3}\,\text{fm}\right)^2 = 0.0144A^{2/3}(\text{b}) \qquad (2)$$

With the atomic mass number of a nucleus A .

$B(E2)\uparrow$ is the reduced electric transition probability from the ground state to the first state for even- even nucleus of the isotopes was 2^{+} counted using the following equation [11]:

$$B(E2)\uparrow = 2.6E^{-1}Z^{2}A^{-2/3}(\mathrm{e^2b^2}) \qquad (3)$$

E is the energy of Gamma ray transition in KeV units, of the first excited state 2^+ was obtained from [7].

The other method for calculation of deformation parameter δ is by using the intrinsic quadrupole moments Q_0 can be written as [12-13]:

$$\delta = \frac{3Q_0}{4Z\langle r^2\rangle} \qquad (4)$$

Where:

$\langle r^2\rangle$ mean squared charge distribution radius average is equal to [14-15]:

$$\langle r^2\rangle = \frac{3}{5}R_0^2 = \frac{3}{5}\left(1.2A^{1/3}\right)^2 \quad (A > 100) \quad (5)$$

The intrinsic electric quadrupole moment Q_0 is given by [16]:

$$Q_0 = \sqrt{\frac{16\pi}{5}\frac{B(E2)\uparrow}{e^2}} \qquad (6)$$

To determine major (a) and minor (b) ellipsoid axises from following equations [15- 17]:

$$a = \sqrt{5\langle r^2\rangle - 2b^2} \qquad (7)$$

$$b = \left[\langle r^2\rangle\left(1.667 - \frac{2\delta}{0.9}\right)\right]^{1/2} \qquad (8)$$

The difference between major and minor of ellipsoid axes can be given as [6, 18- 19]:

$$\Delta R_1 = a - b \qquad (9)$$

$$\Delta R_2 = \frac{\beta}{1.06}R_0 \qquad (10)$$

Where:

R_0 are the nuclear charge radii $R_0 = 1.2A^{1/3}$ [20].

2. Results and Discussions

In this work $B(E2)\uparrow$, β and Q_0 have been calculated from equations (1,3,6) and presented in Tables (1,3), for $_{56}\mathrm{Ba}$ and $_{82}\mathrm{Pb}$ isotopes respectively. $\langle r^2\rangle$ values which calculated by using equation (5) was presented in Tables (2,4) for isotopes under study. The different between (a,b) can be obtained by two methods $\Delta R_1, \Delta R_2$ by equations (9) and (10) respectively, and The results have been presented in Tables 2 and 4, and were plotted as a function to neutrons number N as shown in figures (2) and (3), we noted that the values of different between (a,b) decreased with increasing of N until reach the minimum values at the magic number for barium and lead isotopes respectively. Our results shown that, the values of ΔR_1 is very close to ΔR_2, then we can consider that $\Delta R_1 \approx \Delta R_2 \approx \Delta R$.

The difference between a and b ellipsoid axes ΔR can be help us to determine the (oblate , prolate or spherical) shape of the nucleus, where the standard values of each limit is $\Delta R_{\exp} \geq 0.85$ its shape will be prolate, $0.75 \leq \Delta R_{\exp} < 0.85$ the nucleus will have an oblate shape, while for values of $\Delta R_{\exp} < 0.75$ the nucleus will have a spherical shape [21]. from calculated values of a and b ellipsoid axes and the difference between them we have obtained the shape of nuclides for even- even and isotopes, by plotting (a , b) ellipsoid axes in two dimensions (2D) as shown in figures (4 -9) for $_{56}\mathrm{Ba}$ and $_{82}\mathrm{Pb}$ isotopes respectively.

Table 1. The calculated values of Q_0 and β for $_{56}$Ba isotopes

A	N	E [KeV]	$B(E2)$ [e^{2}b^2]	Q_0 [b]	R_0^2 [b]	β
122	66	196	1.691	4.122	0.354	0.274
124	68	229.91	1.426	3.786	0.358	0.249
126	70	256.02	1.267	3.568	0.362	0.233
128	72	284	1.130	3.370	0.366	0.217
130	74	357.38	0.889	2.989	0.370	0.191
132	76	464.51	0.677	2.608	0.373	0.165
134	78	604.72	0.515	2.275	0.377	0.142
136	80	818.52	0.377	1.945	0.381	0.120
138	82	1435.805	0.213	1.462	0.385	0.090
140	84	602.37	0.502	2.246	0.388	0.136
142	86	359.81	0.833	2.892	0.392	0.174
144	88	199.326	1.489	3.868	0.396	0.231
146	90	181.04	1.624	4.040	0.399	0.239

Table 2. The calculated values of a, b and ΔR with two deferent methods for $_{56}$Ba isotopes

A	N	$\langle r^2 \rangle^{1/2}$	a [fm]	b [fm]	ΔR_1 [fm]	ΔR_2 [fm]
122	66	0.21	7.74	4.81	1.465	1.541
124	68	0.215	7.64	4.95	1.341	1.408
126	70	0.217	7.577	5.058	1.260	1.320
128	72	0.219	7.525	5.153	1.186	1.240
130	74	0.222	7.398	5.298	1.050	1.094
132	76	0.224	7.269	5.438	0.915	0.950
134	78	0.226	7.158	5.563	0.797	0.824
136	80	0.228	7.046	5.683	0.681	0.701
138	82	0.231	6.866	5.841	0.513	0.524
140	84	0.233	7.223	5.670	0.777	0.802
142	86	0.235	7.510	5.531	0.989	1.028
144	88	0.237	7.910	5.297	1.307	1.368
146	90	0.240	7.997	5.283	1.357	1.423

Table 3. The calculated values Q_0 and β for $_{82}$Pb isotopes

A	N	E [KeV]	$B(E2)$ [e^{2}b^2]	Q_0 [b]	R_0^2 [b]	β
200	118	1026.6	0.498	2.237	0.49	0.073
202	120	960.7	0.529	2.305	0.49	0.075
204	122	899.17	0.561	2.374	0.49	0.077
206	124	803.1	0.624	2.504	0.50	0.080
208	126	4085.4	0.122	1.107	0.51	0.035
210	128	799.7	0.619	2.493	0.51	0.079
212	130	804.9	0.611	2.478	0.51	0.078
214	132	836	0.585	2.423	0.52	0.076

Table 4. The calculated values of a, b and ΔR with two deferent methods for $_{82}$Pb isotopes

A	N	$\langle r^2 \rangle^{1/2}$	a [fm]	b [fm]	ΔR_1 [fm]	ΔR_2 [fm]
200	118	0.295	7.637	6.687	0.475	0.484
202	120	0.297	7.676	6.701	0.488	0.497
204	122	0.299	7.716	6.715	0.501	0.511
206	124	0.301	7.771	6.719	0.526	0.537
208	126	0.303	7.418	6.951	0.234	0.237
210	128	0.305	7.810	6.769	0.520	0.531
212	130	0.307	7.826	6.795	0.516	0.526
214	132	0.309	7.833	6.827	0.503	0.513

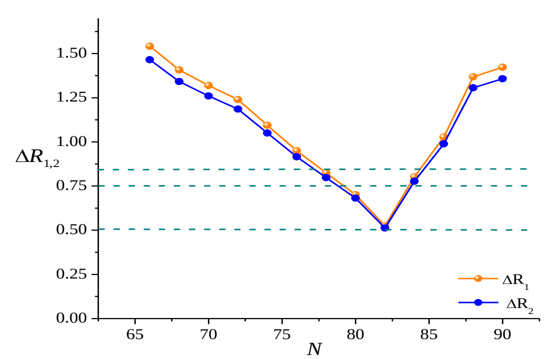

Fig. 2. The difference between (a,b) ellipsoid axes by two methods for $_{56}$Ba isotopes

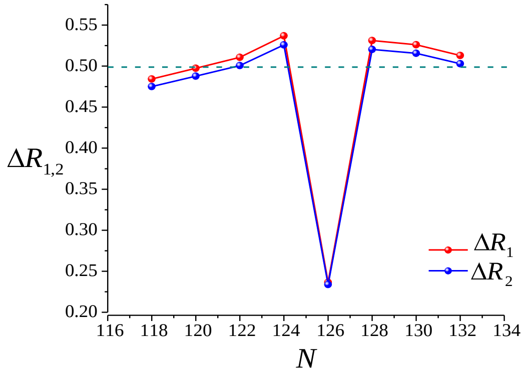

Fig.3. the difference between (a,b) ellipsoid axes by two methods for $_{82}$Pb isotopes

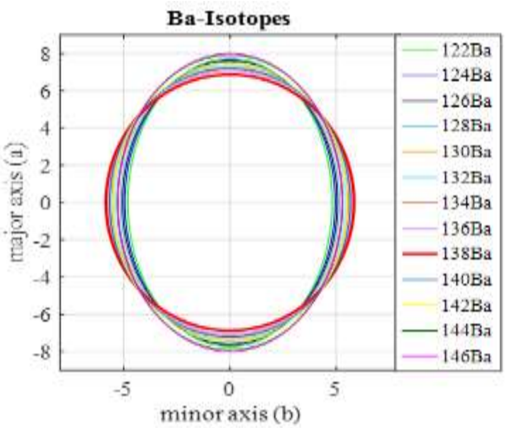

Fig. 4. Evolution of shape with increasing number of neutrons for $_{56}$ Ba isotopes

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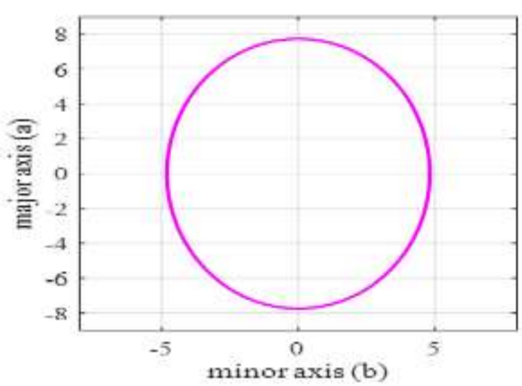

Fig. 5. Prolate shape for ^{122} Ba isotope

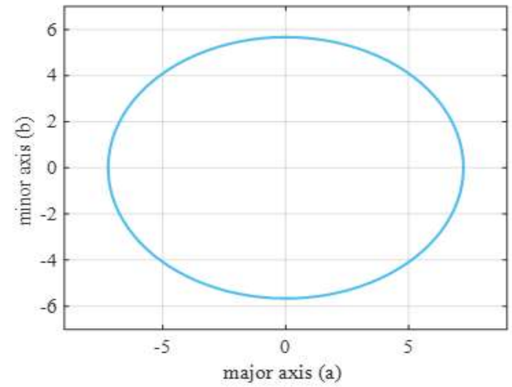

Fig.6. Oblate shape for ^{140} Ba isotope

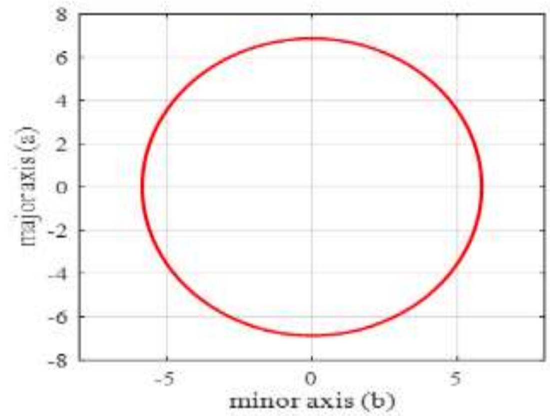

Fig. 7. Spherical shape for ^{138} Ba isotope

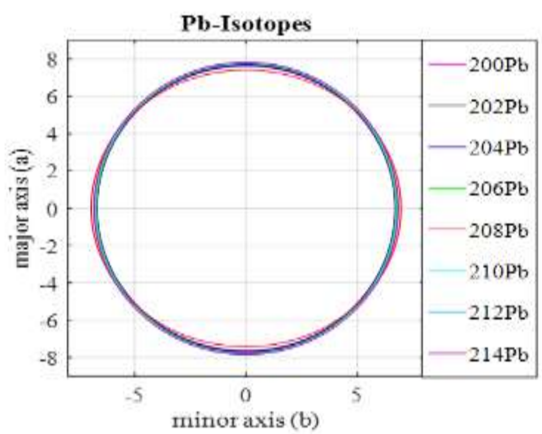

Fig.8. Evolution of shape with increasing number of neutrons for $_{82}$Pb isotopes

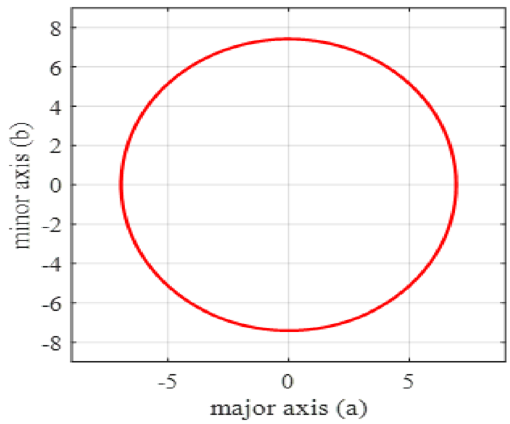

Fig.9. Spherical shape for ^{208}Pb

3. Conclusion

This study is focused on ΔR values When compare our results with the standard values ΔR_{exp} , we noted that all of ${}_{56}$Ba isotopes shape will be deformed shape, except ^{138}Ba isotope which has a spherical shape, and the most of it has prolate shape expect ^{134}Ba and ^{140}Ba which take oblate shape. While the shape of ${}_{82}$Pb nuclei will be spherical in all isotopes, because of $\Delta R < 0.5$ .Finally the reducing values of the deformation parameters β and the intrinsic electric quadrupole moment Q_0 is close to the magic number which means that the shape of nuclei with magic numbers of neutrons having a closed shell (spherical shape) which proofed in reference [22].

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