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Simulation of Electromagnetic Waves Propagating inside Cylinder in Free Space using Finite Difference Time Domain (FDTD) Technique

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Abstract

In this paper, we consider the electromagnetic waves propagating inside empty cylinder by utilizing the finite-difference time-domain (FDTD) technique. The cylinder is made of a perfect electric conductor (PEC) placed in a domain in free space. The FDTD technique utilized in this study for solving Maxwell’s time dependent curl equations for a one, two and three dimensions in order to make a comparison between the simulations when inserting PECs at specific locations in all simulations. The results of simulations demonstrated that the fields calculated as well as controlled to propagate in free space in all dimensions.

Key words: Finite-Difference Time-Domain (FDTD); Maxwell’s equations; Electric fields; Magnetic fields, Perfect Electric Conductor (PEC); One-dimensional (1D FDTD); Two-dimensional (2D FDTD); Three-dimensional (3D FDTD).

المستخلص

في هذه الورقة تم محاكاة الموجات الكهرومغناطيسية المنتشرة داخل أنبوبة مجوفة من موصل مثالي موجود في الفضاء باستخدام طريقة الفروق المحددة. تم التطبيق والمحاكاة لحل معادلات ماكسويل في بعد واحد وبعدين وفي ثلاثة ابعاد والمقارنة عند استخدام الموصل المثالي في كل حالة.لقد تبين من النتائج تحكم في انتشار الموجات الكهرومغناطيسية في جميع الابعاد.

Introduction

There are many methods that can be used to simulate electromagnetic waves propagation. The finite difference time domain (FDTD) is one of the most widely used computational methods to simulate electromagnetic problems. The method was originally introduced to the electromagnetic community by Kane Yee (1966) for three-dimensional solution of Maxwell's time dependent curl equations using central difference approximations of the spatial and temporal derivative [1]. This technique was developed to find solutions of Maxwell's equations for complicated structures. The technique is applied to many applications such as waveguides, electromagnetic absorption in human tissues, antenna and in microwave circuits which are difficult sometimes to obtain an analytical result for.

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The aim of this research is to compute the fields that generate in a cylinder in free space. For that reason the numerical approximation FDTD solution is applied here. The electric and magnetic fields can be computed inside the structure. This is in order to observe the distributions of electromagnetic waves as concentrated in the cylinder by using perfect electric conductor (PECs). The PECs are required in this study to control the propagation of the electromagnetic waves in one, two and three dimensions. In this paper, we will provide a calculation, for example, in one dimension and describe the use of a hard source as a comparison to a soft source. The hard source will be utilized when simulating, for example, two-dimensional and three-dimensional cases. This kind of calculation will be explaining the effect when including a perfect electric conductor in each calculation. Therefore, computer programs were prepared to implement the FDTD for the 1D-FDTD, 2D-FDTD in addition to 3D-FDTD method. This will be displayed in the results section and comparisons between computation dimensions will be presented.

Method

The finite difference time domain (FDTD) method applies the finite difference as approximation to spatial and temporal derivatives. This method is applied in this study to solve Maxwell's curl equations as Ampere's and Faraday's laws that couple the electric and magnetic fields provided by the equations (1.a) and (1.b). The two equations were numerically simulated as will be demonstrated later in the simulation results. For a one dimension model (1D-FDTD), two components require computing in a simulation [2]:

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_o}\frac{\partial H_y}{\partial x} \tag{1.a}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_o}\frac{\partial E_z}{\partial x} \tag{1.b}$$

For two dimensions model (2D-FDTD), it consists of three components [3]:

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_o}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \tag{2.a}$$

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu_o}\frac{\partial E_z}{\partial y} \tag{2.b}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_o}\frac{\partial E_z}{\partial x} \tag{2.c}$$

Finally, from the curl Maxwell equations we can obtain the following equations for three dimensions model (3D-FDTD) which consist of six components [4]:

$$\frac{\partial \mathbf{E_x}}{\partial t} = \frac{1}{\varepsilon_o}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) \tag{3.a}$$

$$\frac{\partial \mathbf{E_y}}{\partial t} = \frac{1}{\varepsilon_o}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) \tag{3.b}$$

$$\frac{\partial \mathbf{E_z}}{\partial t} = \frac{1}{\varepsilon_o}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \tag{3.c}$$

$$\frac{\partial \mathrm{H_x}}{\partial t} = -\frac{1}{\mu_o}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) \tag{3.d}$$

$$\frac{\partial \mathrm{H_y}}{\partial t} = -\frac{1}{\mu_o}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) \tag{3.e}$$

$$\frac{\partial H_z}{\partial t} = -\frac{1}{\mu_o}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) \qquad (3.f)$$

Where:

E= ($E_x(x, y, z, t)$, $E_y(x, y, z, t)$, ($E_z(x, y, z, t)$) is the electric field strength,
H= ($H_x(x, y, z, t)$, $H_y(x, y, z, t)$, $H_z(x, y, z, t)$) is the magnetic field strength (t is the time and *x*, *y*, *z* are the spatial coordinate) and μ_o is the permeability and ε_o is the permittivity of the space. Therefore, there are six coupled equations for a three dimensional simulation as demonstrated in eq. (3). For example, in the *x* direction, the temporal derivative of each field component can be provided by the spatial derivative of two components of other fields as expressed in the following equations:

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_o}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) \qquad (4.a)$$

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu_o}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right) \qquad (4.b)$$

We can rewrite eq. (4) in discrete time and space based on the central difference approximation technique which acquires the updating equations as an example for the *x* components which can be rewritten mathematically as [4]:

$$E_x|^{n+1}_{i+\frac{1}{2},j,k} = E_x|^{n}_{i+1/2,j,k} + \frac{1}{\varepsilon_o}.\left(\frac{\Delta t}{\Delta y}\left(H_z|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} - H_z|^{n+\frac{1}{2}}_{i+\frac{1}{2},j-\frac{1}{2},k}\right) - \frac{\Delta t}{\Delta z}\left(H_y|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_y|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k-\frac{1}{2}}\right)\right) \qquad (5.a)$$

$$H_x|^{n+1/2}_{i,j+1/2,k+1/2} = H_x|^{n-1/2}_{i,j+1/2,k+1/2} + \frac{1}{\mu_o}.\left(\frac{\Delta t}{\Delta y}\left(E_z|^{n}_{i,j,k+1/2} - E_z|^{n}_{i,j+1,k+1/2}\right) - \frac{\Delta t}{\Delta z}\left(E_y|^{n}_{i,j+1/2,k} - E_y|^{n}_{i,j+1/2,k+1}\right)\right) \qquad (5.b)$$

Where the space increment is Δx and the time increment is Δt.

There are similar expressions for the *y* and *z* components that can be written using the same approach. We can see from eq. (5) that the update of each electric component requires the surrounding magnetic field's components in other directions. While, the update of each magnetic component requires the neighbouring electric field's components in other directions as shown in figure 1.

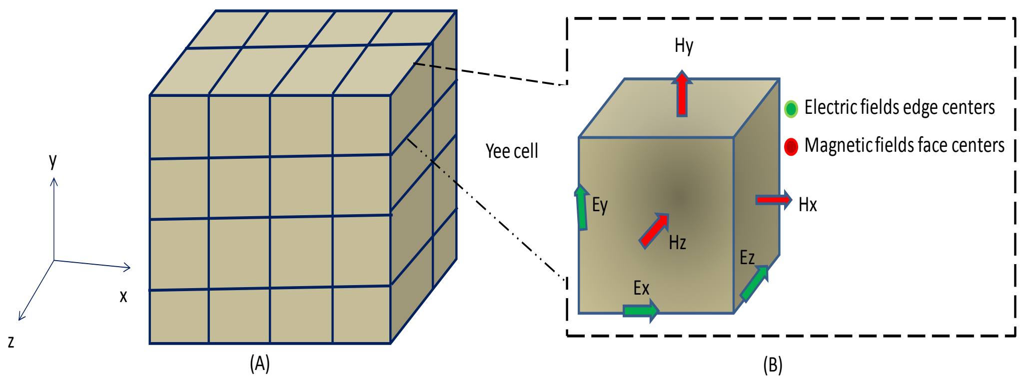

Figure 1. (A) Arrangement of 3D-FDTD by Yee cells and (B) the electric fields are on the cell edges and magnetic fields are on the cell faces.

The MATLAB (R2013a) program language is used in this study in order to solve Maxwell's curl coupled equations based on the components mentioned in the equations (1, 2 and 3). For 3-D as an example, the equations approximated as discrete Yee cells to form three dimensions space as demonstrated in eq. (5). The model described in Figure 2 is implemented in the MATLAB (R2013a) programs based on discrete Yee cells and we begin by constructing the ring and cylinder in two and three dimensions as a mesh mode. The grid dimensions were utilized in 2D-FDTD as 100×100 in the x and y directions while in 3D-FDTD as $100 \times 100 \times 100$ in the x, y and z directions. We will solve the model by generating a mesh in three directions by applying a uniform spatial step size which is used as $(\Delta x = \Delta y = \Delta z = 0.5\ cm)$ along each Cartesians axis. Therefore, the domain should be large enough to model the geometry of interest and each Voxel in the computational domain is filled by the properties of free space and PECs. The PECs can be applied to model structure when no EM signals can appear outside the geometry of interest. Moreover, the cover was added in the *x*-*y* plane on the left side of this cylinder which is also made of a Perfect electric conductor. The PECs of the cylinder were set in the code based on the diameter and length which is about 30 cells in the *x* and 70 cells in the *z* directions. This arrangement will describe the cylinder as a mesh mode as illustrated in Figure 2 and the surrounding media is a free space. Furthermore, there is a number of different display options that can be utilized for displaying data such as in the *x*-*y* and *x*-*z* planes as illustrated in Figure 2.

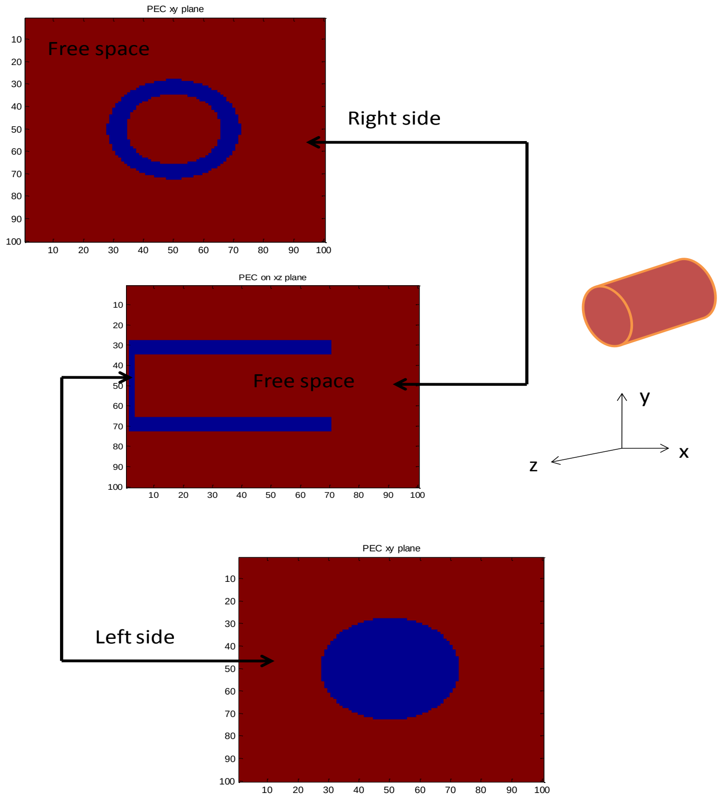

Figure 2. Geometry of the cylinder. The objects presented as mesh mode filled in the domain with the PECs and placed along the *z*- direction in the 3D-FDTD.

Results and Discussion

We considered three cases in this paper and then demonstrate the relationship between the simulations results which will be described and displayed as slices. With regard to the source utilized in this research, the domain can be excited by a Gaussian source that can be applied and set as a hard source which is placed in the middle of the computational domain in each example.

We simulated an electromagnetic signal that generates in free space as in one dimension, an example, in order to understand the effect of applying the PECs and also explain a hard source. Therefore, this study attempts to provide the performance of hard source and the effect of the PECs that have been illustrated in Figure 3 at different time instants. In the following simulations, the line in 1D-FDTD is subdivided into three regions to observe the effect when adding PECs on the electromagnetic waves. On the left side is region one and right side is region two. While a third region is located between the PECs. The PECs were set at a node 20 in region 1, and at a node 80 in region 2 and the third region set as a free space. The signals generated in a domain as demonstrated in Figure 3.A and travel one spatial step in each time step until reaching the nodes where the PECs is located (Figure 3.B). This example demonstrates that the signals when hitting these regions must be reflect back in the opposite directions as shown in snapshots at 100 and 130 time steps in Figs. 3.C and D, respectively. Moreover, this result is significant. The snapshot at 130 time steps, in Figure 3.D, shows that the signal is reflected completely. The results of simulation can explain that the electric field is inverted while at the same time the magnetic field is not inverted as shown in Figure 3.D.

Moreover, the simulation result shown in Figure 4 demonstrated that the source is placed at the spatial position x=35 and the electric and magnetic fields produced to propagates in both direction are propagating bilaterally in the positive and negative directions. It can be noted that from Figure 4.A the signal generated by a hard source and updated in a computational domain until reaching a PEC. The signal reflected back on the right side and also when reaching at a point of source location the signal reflected on left side. It means that the left portion is localized between the PEC and at a node of hard source. However, the signal generated by a soft source also reflected back when hitting a PEC on the left side and then propagated to the positive direction and passed through a node of source location as shown in Figure 4.B. Therefore, we can clearly say that a hard source has acted as a metal at a node of the source location.

Up to this point one dimension case is simulated after that it can be simulated in the fields in two and three dimensions by using the same approach.

The second case can be studied in 2D model, the shape is constructed as a ring in the x and y directions shown in Figure 5.A. Therefore, the simulation generates three components such as the H_x, H_y, and E_z and the components will update in the domain inside the ring shape in free space in each pixel. It can be observed that no signal appeared inside the PEC or outside it as shown in Figure 5.B that have already been noted when simulating 1D as demonstrated in Figure 3. The source located at node (50, 50) and the signal generated in the middle of the grid in two dimensions, the snapshots are provided in Figure 5 at the same time as in three

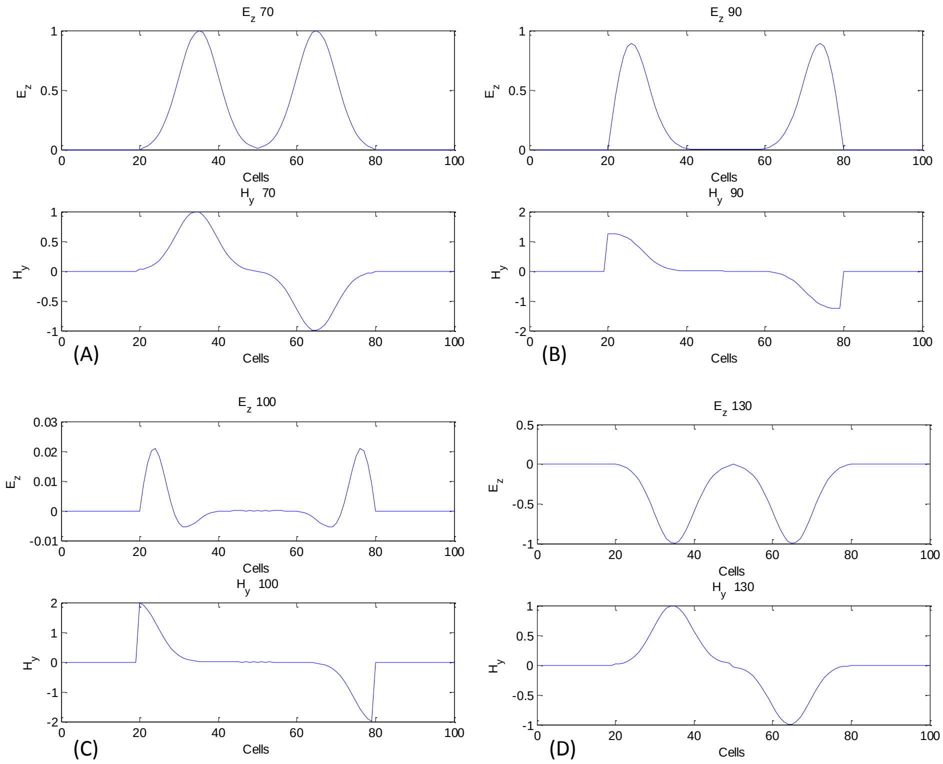

Figure 3. Results of simulation using one-dimensional FDTD. The E_z and H_y produced and propagated along the x axis and generated between the PECs (A), signal is hitting the PECs (B), the signals start reflecting at the both sides (C) and the signals reflected back to the opposite directions (D).

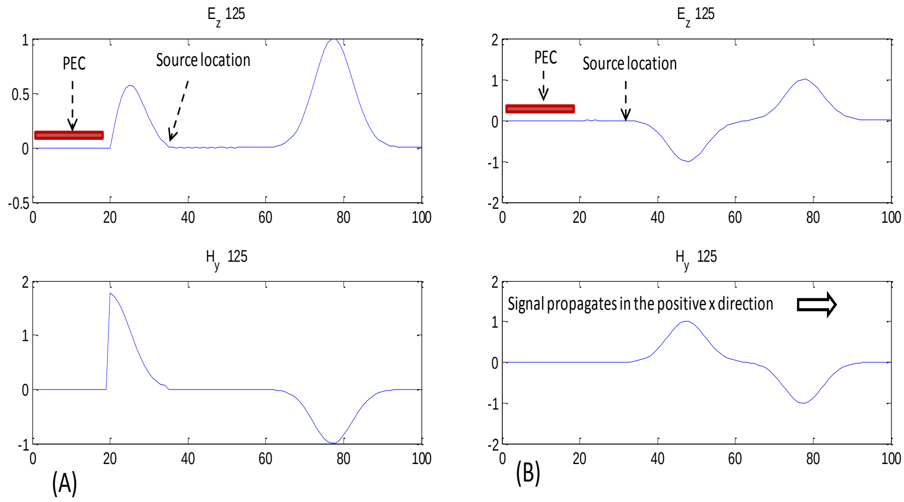

Figure 4. Results of the simulations using one-dimensional FDTD: the signal generated by a hard source (A) and by a soft source (B). The snapshots are taken at 125 time step and then the signals are propagating in the positive x direction:

dimensions, the signal produced in the middle in the *x-y* at the left side in the *z*-direction. This is illustrated in Figs. 6, 7 and 8. Therefore, electromagnetic fields propagated into the positive *z* direction as demonstrated in Figure 8 that displays the snapshots of *Ex*, *Ey*, *Ez, Hx, Hy*, and *Hz* which are taken at 400 time steps. Numerical results stated that the field in a cylinder is oriented in the *z* direction and this verifies that waves propagated inside the structure and flowed in *z* direction. It means that the fields have been concentrated in the regions of interest. The result of the simulation showed that all electromagnetic components generated inside the cylinder only and no signal appeared outside the structure in the images until the end of the cylinder. It means that the fields updated in the three directions in each Voxel at the same time each Voxel represented the values of six components. Therefore, electromagnetic wave transmitted from a source and the performance of using PECs has been evaluated by the FDTD method. It can be noted from the fields demonstrated in Figs 5 and 6 which express the relationship when applying perfect electric conductors in the 2-D and 3-D. It was found that the output of calculations when simulating systems in the two dimensions and three dimensions are quite similar to that modelled in one dimension. The images in Fig. 5 and Fig. 6 demonstrate a comparison of FDTD results in the snapshots computed in 2D-FDTD and 3D-FDTD simulations, respectively. Therefore, it can be said that for two different models, there are similarities produced in the field distributions. This agrees very well with the results obtained when simulating the signal in one dimension in free space and adds the PECs in the end of the line in both sides. The waves kept between the nodes where the locations of PECs and there is no signal penetrated in these locations and with 2-D when the signal generated between the PECs inside the ring.

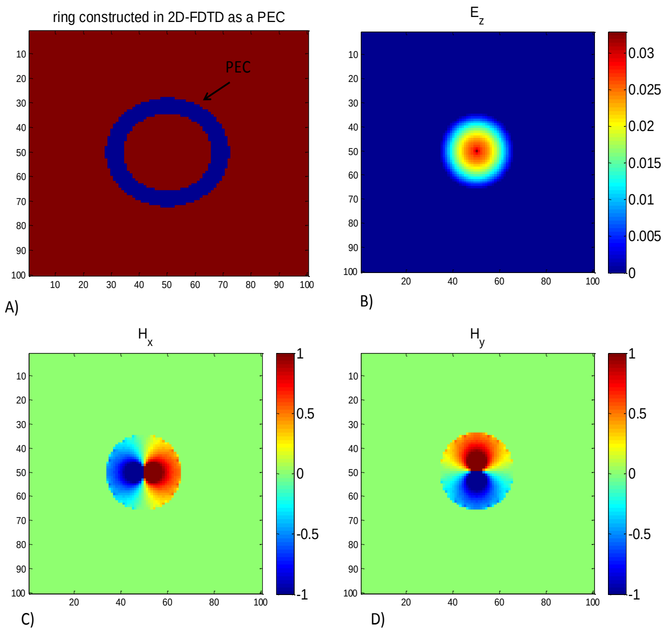

Figure 5. Results of a simulation using two-dimensional FDTD. The electromagnetic waves are produced inside a ring made of a PEC in a free space.

Moreover, Figs. 6, 7 and 8 demonstrate snapshots of the electric and magnetic fields components in the *x*-*y*, *x*-*z* and *y*-*z* planes, respectively. In the present paper, it is interesting to observe that how the PECs influence the waves to move between the perfect electric conductors inside the cylinder in the positive *z* direction that excited by the hard source placed on the left side, the waves propagated until reaching the end of the cylinder and then the waves spread out through a free space. We can now say that the results obtained from the FDTD for the 1D and 3D indicated that the directions of the wave's propagation controlled both in the positive *x* and *z*-directions, respectively.

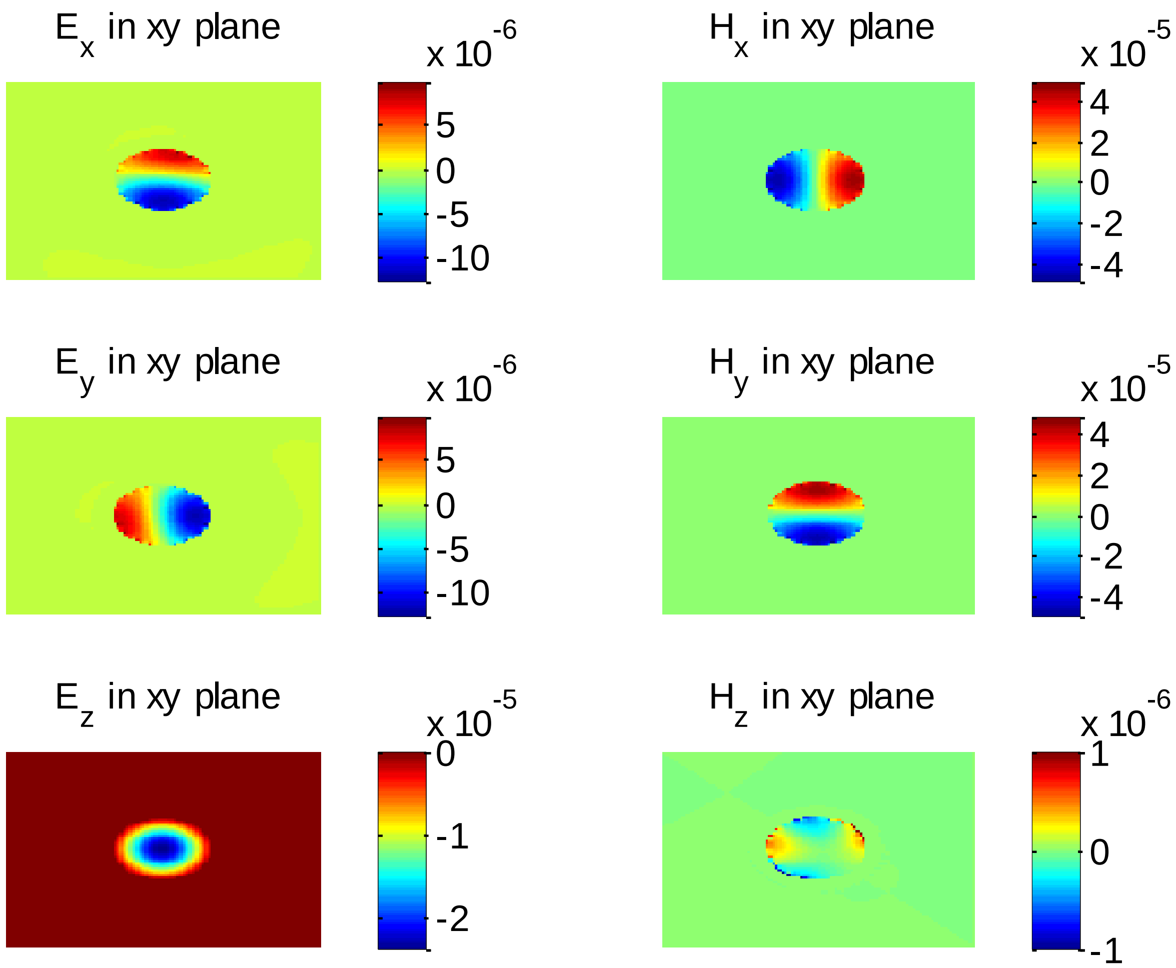

Figure 6. Results of simulation using three-dimensional FDTD. The electric and magnetic fields components in the *x*-*y* plane are generated at 400 time steps.

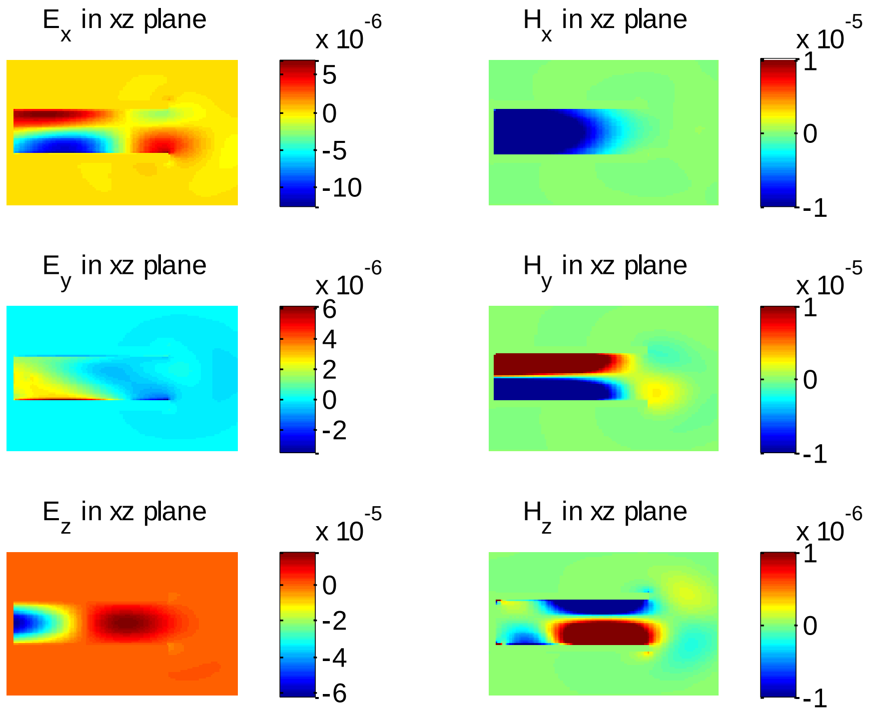

Figure 7. The electric and magnetic components in the *x*-*z* plane at 400 time steps in 3D-FDTD.

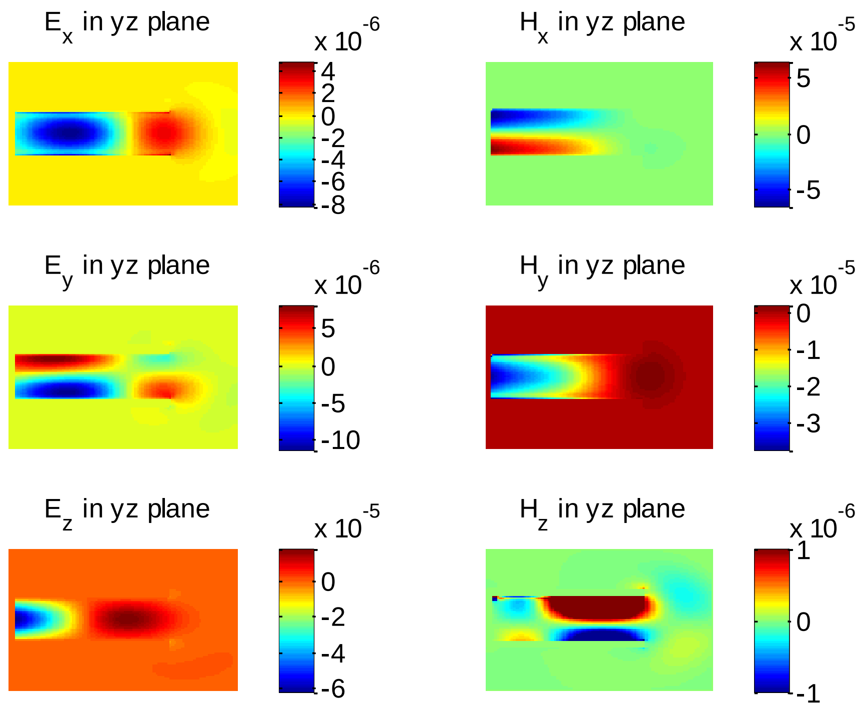

Figure 8. The electric and magnetic components in the *y*-*z* plane at 400 time steps in 3D-FDTD.

Conclusion

We have demonstrated that the FDTD technique can be utilized to generate the electromagnetic waves for example inside a cylinder shape. The method is an extremely good technique to model very complicated structures. In this study, this method was utilized to solve 1D-FDTD, 2D-FDTD and 3D-FDTD simulations which are very hard sometimes to find the exact solutions for analytically. A significant observation is that the domains were excited by applying a hard source which is a simple source that generated very good homogeneous distributions.

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