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Electric Quadrupole Moment of Even-Even Nuclei

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Abstract

In the framework of hydrodynamic model, a new calculations of the electric quadrupole moment of even-even nuclei has been obtained using the first excited state energy of rotational band. With the assumption that nuclei are volume preserving under rotation, the radii of spheroid have also been computed. In this work, the isotope of Dy164 showed a noticeable difference in size.

Keywords: Inertia; Charge; Quadrupole; Rotation; Spheroid; Spin.

المستخلص

في إطار النموذج الهيدروديناميكي ، أجريت حسابات جديدة للعزم الكهربي رباعي القطب للانوية الزوجية– الزوجية من خلال توظيف مستوى الاثارة الأول في النطاق الدوراني ، وذلك بفرض أن الانوية ثابته الحجم أثناء عملية الدوران حيث تم أيضا حساب أنصاف أقطار شبه الكرة . في هذا البحث أظهر نضير Dy164 اختلافا ملحوظا في الحجم.

Introduction

A nucleus with unpaired nucleons will have a charge distribution which results in an electric quadrupole moment. Properties of these nuclei with several nucleons outside a closed shell are described in a first approximation by their interactions with an inert core plus other nucleons which can interact with the core and mutually with each other via a residual interaction [1], [2].

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The allowed nuclear energy levels are shifted unequally due to the interaction of the nuclear charge with an electric field supplied by the non-uniform charge distribution [3]. One of the main causes of nuclear deformation is made apparent by connection with electric quadrupole moments [4], together with the low energy spectrum contains sequences of rotational states varying with quantized angular momentum [5].

Nuclear Electric Quadrupole Moment

An even-even nucleus with mass M in the region $150 < A < 190$ can rotate about an axis at right angles to the axis of symmetry, forming an axially-symmetric rigid rotator with uniform mass distribution [6], these rotations can only be observed in nuclei with non-spherical equilibrium [7], [8]. The energy spectrum of such rotator with moment of inertia ϑ and quantized angular momentum I is given by [9]

$$E_I = \frac{\hbar^2}{2\vartheta} I(I+1) \tag{1}$$

with the moment of inertia is taken to be a classical spheroid of rotation

$$\vartheta = \frac{1}{5} M(a^2 + c^2) \tag{2}$$

where a denotes the distance along the axis of symmetry and c is the distance along the axis perpendicular to the axis of symmetry. The quantum analogue of the moment of inertia can be obtained from relation [10]

$$\vartheta = \frac{\hbar^2}{2} \left(\frac{dE}{dI(I+1)} \right)^{-1} \tag{3}$$

By definition, the intrinsic quadrupole moment, $\int d^3r\, \rho r^2 (3\cos^2\theta - 1)$ [11], which is evaluated for the case of an axially-symmetric rigid rotator of total number of charges Z uniformly distributed

$$Q_0 = \frac{2}{5} Z(a^2 - c^2)\ . \tag{4}$$

For the case of well deformed axially symmetric nuclei, the measured quadrupole moment Q can be related to the intrinsic quadrupole moment [12]

$$Q = Q_0 \frac{3K^2 - I(I+1)}{(2I+3)(I+1)} \tag{5}$$

where K is the projection of total nuclear spin I onto the axis of symmetry, $K = 0$ for even-even axially symmetric nuclei [13].

Table 1. The units of the ground state energy E_{2^+} is given in (keV), spheroid volume a^2c in (fm^3), and quadrupole moment $Q(E_{2^+})$ in (b). The observed values listed in last column with no sign if it was not determined by experiment [14].

Nucl	A	Z	E_{2^+}	a^2c	Q_{cal}	Q_{exp}
Gd	154	64	123	193	−1.80	−1.82(4)
	156		89	254	−1.96	−1.93(4)
	158		80	271	−2.00	−2.01(4)
	160		75	240	−2.08	−2.08(4)
Dy	160	66	87	243	1.89	1.8(4)
	164		73	574	−2.08	−2.08(15)
Er	166	68	81	132	−2.70	−2.7(9)
	170		79	274	−1.91	−1.9(2)
Yb	170	70	84	218	2.20	2.1(4)
	174		77	267	2.18	2.1(3)
	176		82	221	2.22	2.2(4)
Hf	176	72	88	242	−2.04	−2.01(2)
	178		93	266	−2.02	−2.02(2)
W	182	74	100	217	−2.11	−2.1(4)
	184		111	202	−1.92	−1.9(2)
	186		123	189	−1.61	−1.6(3)

Results

Under the assumption that the deformation is volume preserving i.e., $a^2 \propto c^{-1}$, and employing Eq. (2) the radii of spheroid were evaluated by means of any suitable roots finding algorithm [15]. Hence the quadrupole moment for a number of deformed even-even nuclei can be computed using the measured energy of the first excited state E_{2^+}, corresponding to ground state rotational band $K = 0$. Values so obtained are shown in Table 1.

Conclusion

We conclude that in the framework of the nuclear collective model, the quadrupole moments of a number of permanent deformed nuclei were calculated successfully. The obtained values of electric quadrupole moment show good agreement compared with earlier suggested references. Spheroid radii of deformed nuclei in question also obtained. Under the same constraints, we noticed small stretching in volume of listed nuclei when compared with the size of their constituent nucleons. On the other hand, an exception has been noticed in Dy164 which registered quite large deviation. We believe that the calculations carried out are sensitive, and that the nuclear radius constant may conveniently adjusted. Although the odd behavior of Dy164 is surprising and would immediately raise a question.

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