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Application of Finite Difference Time Domain (FDTD) Method to Calculate Poynting Vector of EM Waves in Free Space

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Abstract

In this paper, the implements of a numerical approach utilizing the finite difference time domain (FDTD) method was applied in three dimensional electromagnetic wave formulations and the computational domain can be updated explicitly. This calculation based on solving Maxwell's curl equations by utilizing the FDTD. The space has been excited for a short duration by three sinusoidal signals at the centre of the space and in three different locations. To allow for comparison between the models, the signals generated by means of a variety of positions. This study showed that the Poynting vector components intensities had changed due to the addition of sources in the space. It was noted that Poynting vector image intensities were variances which depend on the location of the excitation pulse. Dependent on the position of the source whether it is close in proximity or spread apart from one another. Therefore, the greater the source distribution the larger volume covered in the space.

Key words: electromagnetic; Poynting vector; Finite Difference Time Domain (FDTD) method.

المستخلص

تم في هذه الورقة محاكاة الموجات الكهرومغناطيسية المنتشرة فى الفضاء حيث تم حل معادلات ماكسويل في ثلاثة أبعاد باستخدام طريقة الفروق المحددة حيث تم تحديث الفضاء بطريقة الفروق المحددة صريحة. تم توليد الموجات الكهرومغناطيسية في الفضاء بواسطة استخدام ثلاثة مصادر تبعث موجات جيبية. تم وضع هذه مصادر في ثلاثة مناطق مختلفة حتى نستطيع المقارنة عند تغير الموقع لكل مصدر على توزيع الموجي. تم في هذه الدراسة حساب متجه بوينتك في ثلاث أبعاد وقد أظهرت المحاكاة عند تغير مواقع المصادر يتأثر توزيع متجه بوينتك. حيث لوحظ عند توزيع أكثر من مصدر تتم تغطيه في حجم كبير.

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Introduction

Maxwell's equations provide a description of EM phenomena. In some cases, exact solution is difficult or impossible to solve analytically. An alternative approach is numerical techniques that should be applied to acquire the approximate solutions. The numerical method as outlined in this research is a time domain approach. Examination of the arrangements of the excitations pulses in the space will be considered. The arrangement of the sources in space may be such that the excitations add up to give a radiation energy maximum in an exacting direction or minimum in others. Therefore, the following components of Poynting (S_x, S_y and S_z) should be computed. These arrangements of sources in a mesh that are explained in the results section resulted in the increase or decrease of the energy in some locations in the girds.

Method

The purpose of any numerical technique in the field of electromagnetic is to find approximate solutions to Maxwell's equations. We can determine the accurate value of the electric and magnetic fields by utilizing these equations. A number of methods have been developed to solve electromagnetic problems such as the Method of Moment (MoM), the Finite Element method (FEM) and Finite-Difference method. The first two methods are implemented through the frequency domain. However, the numerical technique utilized in this research is a time domain approach. The FDTD was originally proposed by Yee S. Kane in 1966 [1], and then improved by others. This method provides an efficient way of solving Maxwell's time dependent curl equations and a computational intensive simulation approach. Its uses include modelling a variety of types of electromagnetic problems such as antenna, waveguides, electromagnetic absorption in human tissues (bio-electromagnetic) and microwave circuits. We used this method to solve Maxwell coupled equations in free space [2]:

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0}\nabla \times \mathbf{E}, \tag{1}$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{t}} = \frac{1}{\varepsilon_0}\nabla \times \mathbf{H} \tag{2}$$

Where μ_o is the permeability and ε_o is the permittivity of free space.
We can write Eq. (1) and Eq. (2) in the rectangular co-ordinate (x, y, z) as the following [3]:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu_o}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right) \tag{3}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_o}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) \tag{4}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_o}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) \quad (5)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon_o}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) \quad (6)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu_o}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \quad (7)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_o}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \quad (8)$$

To form a three dimensional volume structure, many FDTD cells ($N_x \times N_y \times N_z$) should be combined together. This method depends on Yee cell. It showed in Fig. 1 that has the following features: The electric field is defined at the edge centres of a cube and the magnetic field at the face centres.

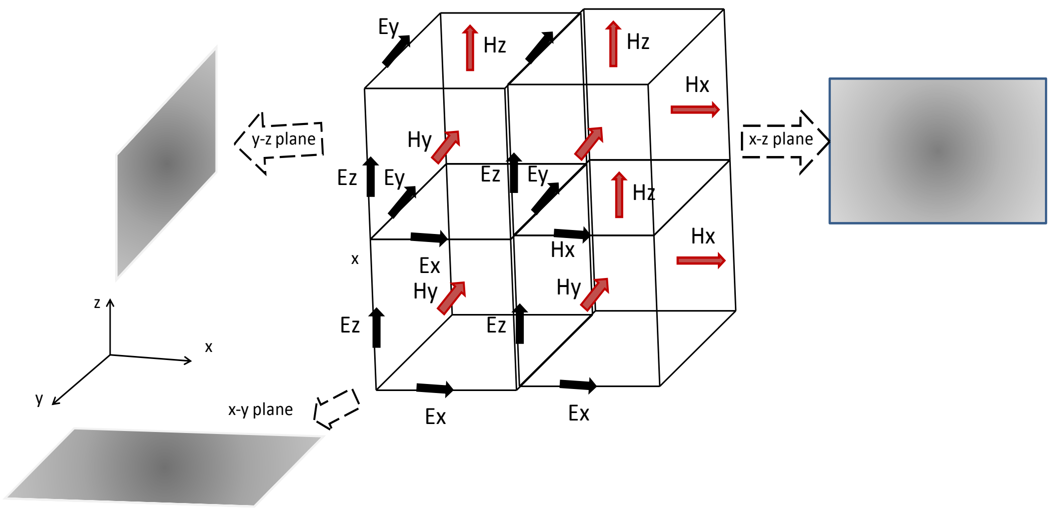

Figure 1. A standard Cartesian Yee cell: Electric field edges centres and magnetic fields face centres). All cells added together to form 3D volume [3].

We can obtain 3D FDTD formulations by applying the central finite difference approximation to all derivatives. 3D formulations are used in this research for time varying electromagnetic in the space. The FDTD update equations can be obtained from the approximation of Faraday's and Ampere's laws. We obtained the explicit finite difference approximation of Eq. (3) and Eq. (4) for the *x* component [3]:

$$E_x^{\,n+1}\left(i+\tfrac{1}{2},j,k\right) = E_x^{\,n}\left(i+\tfrac{1}{2},j,k\right) + \frac{\Delta t}{\varepsilon_0\,\Delta x}\Big(H_z^{\,n+\frac{1}{2}}\left(i+\tfrac{1}{2},j+\tfrac{1}{2},k\right) - H_z^{\,n+\frac{1}{2}}\left(i+\tfrac{1}{2},j-\tfrac{1}{2},k\right) + H_y^{\,n+\frac{1}{2}}\left(i+\tfrac{1}{2},j,k-\tfrac{1}{2}\right) - H_y^{\,n+\frac{1}{2}}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right)\Big) \quad (9)$$

$$H_x^{n+\frac{1}{2}}\left(i,j+\tfrac{1}{2},k+\tfrac{1}{2}\right)=H_x^{n-\frac{1}{2}}\left(i,j+\tfrac{1}{2},k+\tfrac{1}{2}\right)+\frac{\Delta t}{\mu_0\,\Delta x}[E_y^{\,n}\left(i,j+\tfrac{1}{2},k+1\right)-E_y^{\,n}\left(i,j+\tfrac{1}{2},k\right)+E_z^{\,n}\left(i,j,k+\tfrac{1}{2}\right)-E_z^{\,n}\left(i,j+1,k+\tfrac{1}{2}\right) \quad (10)$$

Where the subscript defines the coordinates of the vector and the superscript defines the time step. The factor (1/2) in the above equations refers to the half cell and a half time step. There are equations for y and z field components. These equations are known as update equations and can be implemented in a computer program. These equations imply that an electromagnetic wave propagating in free space with the speed of light. The equations demonstrate that the future value of E depends on its pervious value and the neighbouring magnetic fields. Similar to the update equations for the magnetic field, the future value of H depends on its past value and the value of the neighbouring electric fields. The solution of an electromagnetic problem requires finding the electric and magnetic fields. These components can be used to compute the Poynting vector which is defined by [4]:

$$\mathbf{S}=(\mathbf{E}\times\mathbf{H}) \quad (11)$$

The importance of Eqs. (9-10) is that they provide the future values by the surrounding points in space. The Poynting vector components (S_x, S_y and S_z) were computed utilizing the surrounding electric and also magnetic fields components. The above equations were used to compute these components in three directions and demonstrated as images in the *x, y* and *z* planes. The energy was computed in each voxel in 3D. This calculation establishes the directional energy flux density of an electromagnetic field.

Furthermore, focus was placed on the simulation of electromagnetic waves generated by means of three signals emitting sources in the same phase. This allowed for the assessment of the result of combining the signals distributions of the electromagnetic energy. The signals were combined together in each voxel and at each time step. The final results will appear in the images as generated by only one source in space. We have used the signal $(sin(2\pi f\times t))$ as the source generated the fields. A frequency of one GHz was used. 3D study was solved on a *50-by-50-by-50* cube cells with unit cell size is one tenth of the wavelength. We used the size of all cubes to be similar in this study. In this method, the cell size $(\Delta x,\Delta y,\Delta z)$ was calculated with respect to a minimum wavelength which is one tenth of the wavelength [5]. Each cell size requires two time steps for the signal to travel one cell in 3D simulation. The signals were generated in the middle of the grids for duration of 1.3 *ns*. It was realised that including three sources in the FDTD program would result in the increase of the storage arrays.

Moreover, the sources were assigned as values to E_x. These the sources were pointing in the x directions and placed in the same cell in the middle of the grid and also placed in three locations as shown in figure 2. Each point source was surrounded by eight cubes in the grids. In the first model, the sources of excitation are placed in the centre of FDTD domain at the point ($N_x/2$, $N_y/2$, $N_z/2$). The fields can be generated by combining three excited sources in the program and placing them overlapping each other. Moreover, in the second, third and fourth models the sources were placed at the three points as shown in Table 1 and Fig. 2. It can be noted that the sources were placed in two cells, four cells and six cells apart. These numbers are equivalent to *0.2 λ and 0.4 λ and 0.6 λ* respectively.

Table 1. Three sources located at many locations.

model	Position in 3D
1	$(\frac{Nx}{2},\frac{Ny}{2},\frac{Nz}{2})$, $\left(\frac{Nx}{2},\frac{Ny}{2},\frac{Nz}{2}\right)$ and $(\frac{Nx}{2},\frac{Ny}{2},\frac{Nz}{2})$
2	$(\frac{Nx}{2}-2,\frac{Ny}{2},\frac{Nz}{2})$, $\left(\frac{Nx}{2},\frac{Ny}{2},\frac{Nz}{2}\right)$ and $(\frac{Nx}{2}+2,\frac{Ny}{2},\frac{Nz}{2})$
3	$(\frac{Nx}{2}-4,\frac{Ny}{2},\frac{Nz}{2})$, $\left(\frac{Nx}{2},\frac{Ny}{2},\frac{Nz}{2}\right)$ and $(\frac{Nx}{2}+4,\frac{Ny}{2},\frac{Nz}{2})$
4	$(\frac{Nx}{2}-6,\frac{Ny}{2},\frac{Nz}{2})$, $\left(\frac{Nx}{2},\frac{Ny}{2},\frac{Nz}{2}\right)$ and $(\frac{Nx}{2}+6,\frac{Ny}{2},\frac{Nz}{2})$

Results and Discussion

These calculations were computed by using MATLAB 7.10 (R2010a). A computer program was written to calculate the Poynting vector in three directions (x, y and z) that were simultaneously produced as a result of the three sources. The electric and magnetic fields were generated in space by utilizing the coupled Maxwell's equations as mentioned above.

The FDTD approximation and formulations were adopted for this work in addition to using the same parameters and grids for calculating all models. The sinusoidal waves expanded outward from the centre of the space as shown in images below behaving as very small dipoles emitting signals in the space as shown in fig 3. A number of sources can be placed within in the space that can be utilized to generate a dipole. With each iteration in the program, the components are computed in each voxel. This entails that the generation of an electric field consequently generates a magnetic field. Maxwell's equations state that each electric field component has an associated magnetic field component. It means that the time derivative of the electric field is dependent on the curl of the magnetic field.

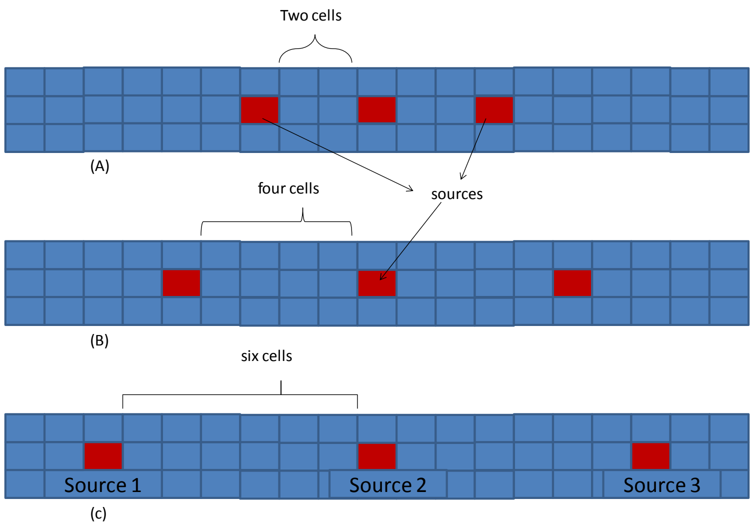

Figure 2. Three sources arranged in the space in three different ways.

The output of the simulation is the S_x, S_y, S_z and S_T at each voxel within the computational domain. We arranged the sources locations as source 1, source 2 and source 3 as shown in fig 2. We can make a comparison based on this armament. The results for S_x, S_y, S_z and S_T are shown in Fig. 3 where the components of electromagnetic waves generated at the same location in the centre across 50 by 50 by 50 grids. All excitations placed in the position of the source 2, it means that the sources overlapped. There are a wide variety of excitations locations used such as in two cells, four cells and six cells apart as shown in fig 2. We placed the first source overlapped with the second and repeated the same arrangements as four cells and six cells apart. The Poynting vector components can be changed by varying the distance between the sources such as $0.2\ \lambda$ and $0.4\ \lambda$ and $0.6\ \lambda$. Figs (4-12) provide images of the radiation patterns and the effect of changing the source position.

The Poynting vector components have different patterns, especially when exciting at two, four and six cells away from the centre domain. When the final calculations in each step were combined together the result demonstrated that the signal that were emitting from different locations behaved out of phase from one another. Moreover, the results have been shown in Figs (4-12) generated by three sources in different locations. It can be noted that the images intensities were affected when changing the positions and also the distances between the sources as shown in Figs (4-12). The results that show the difference when placed source 1, source 2 and source 3 in the same location at ($N_x/2$, $N_y/2$, $N_z/2$) in comparison to the sources placed in two and three locations such as two cells apart and source 1 placed overlapped with middle as in Figs (4-6), four cells apart and source 1 placed overlapped with middle as in Figs (7-9) and six cells apart as in Figs (10-12).

The components generated in the middle of the grids. The advantage of this simulation was being able to observe the Poynting vector behaviour in this area, because we want to make a comparison in this region. The results obtained from the simulations showed that the components affected in term of distributions as waves interfere constructively and destructively with each other in some voxels. Therefore, the components will be influenced when placing the sources apart from each other and many patterns appeared in the simulations.

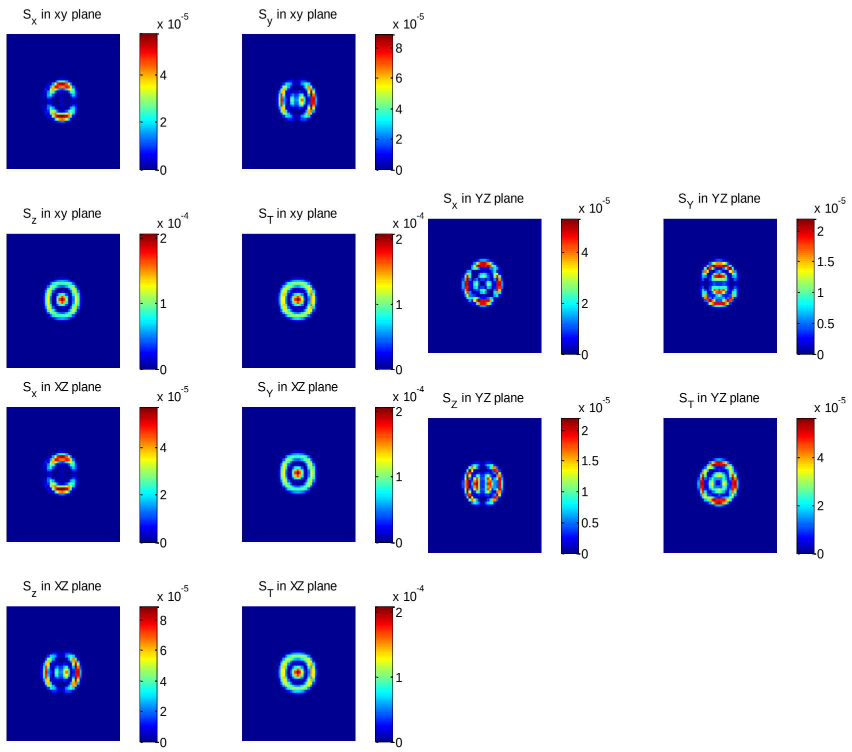

Figure 3. Images in the *x*, y and *z* plane which demonstrate the *Sx*, *Sy*, *Sz* and S_T. Three sources placed overlapped with each other in the middle of the grid.

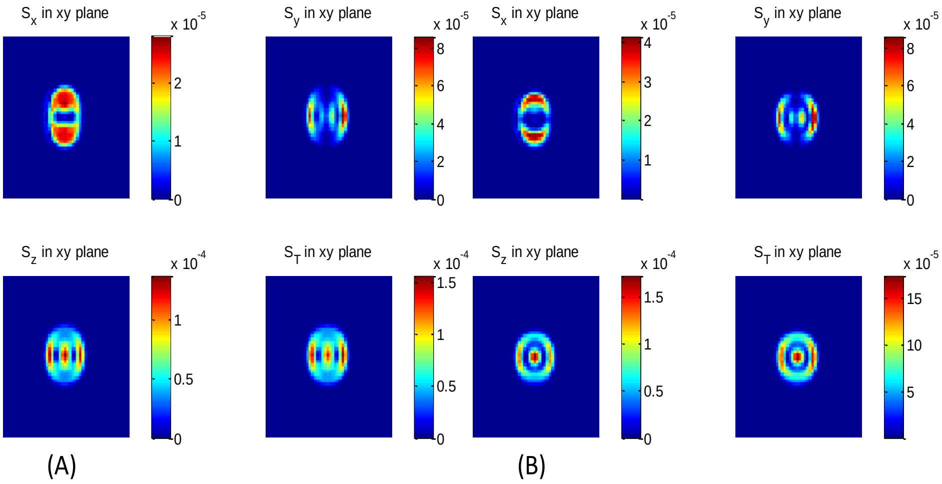

Figure 4. Images in the z plane which demonstrate the *Sx*, *Sy*, *Sz* and S_T. Three sources located in three locations: two cells apart (A) and source 1 placed overlapped with middle (B).

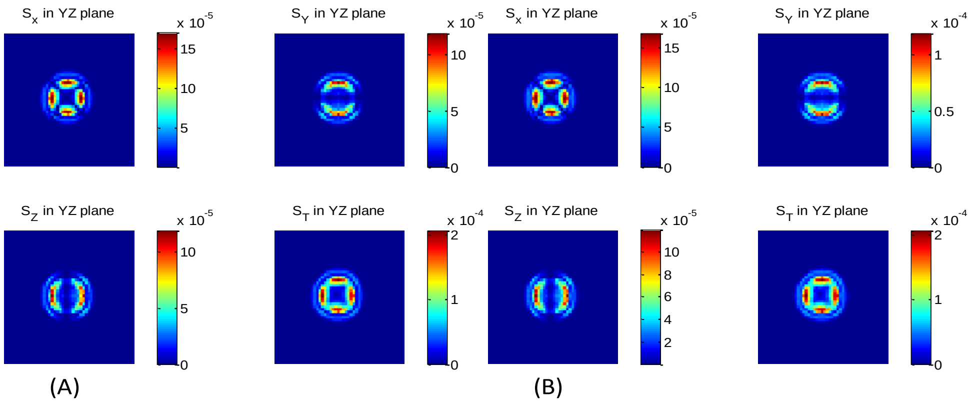

Figure 5. Images in the x plane which demonstrate the *Sx*, *Sy*, *Sz* and S_T. Three sources located in three locations: two cells apart (A) and source 1 placed overlapped with middle (B).

Conclusion

This research illustrates that the methodology examined in this study could be used for modeling of point sources emitting electromagnetic waves utilizing three excitations signals simultaneously. This method provides a simplified manner in which to calculate and solve electromagnetic problems that difficult with analytical method. As presented in the images each voxel generates three components of Poynting vector. Moreover, a variety of simulations were performed to compare the final results. It was noted that the distributions of these components were affected when emitting three signals from different locations. This was observed when changing the position of the sources in the x direction. Therefore, it can be concluded that the FDTD method is one of the key simulation implements in consider of EM propagation.

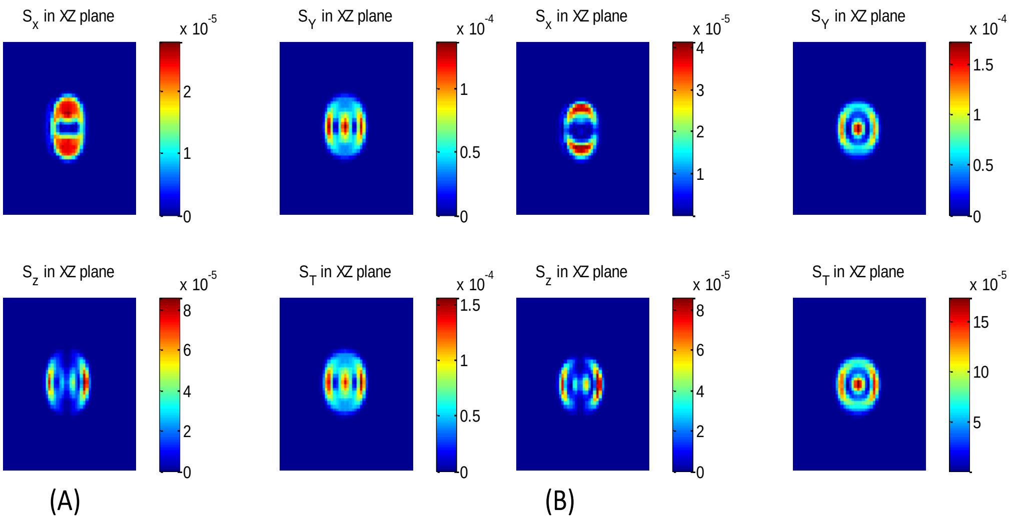

Figure 6. Images in the y plane which demonstrate the *Sx, Sy, Sz* and S_T. Three sources located in three locations: two cells apart (A) and source 1 placed overlapped with middle (B).

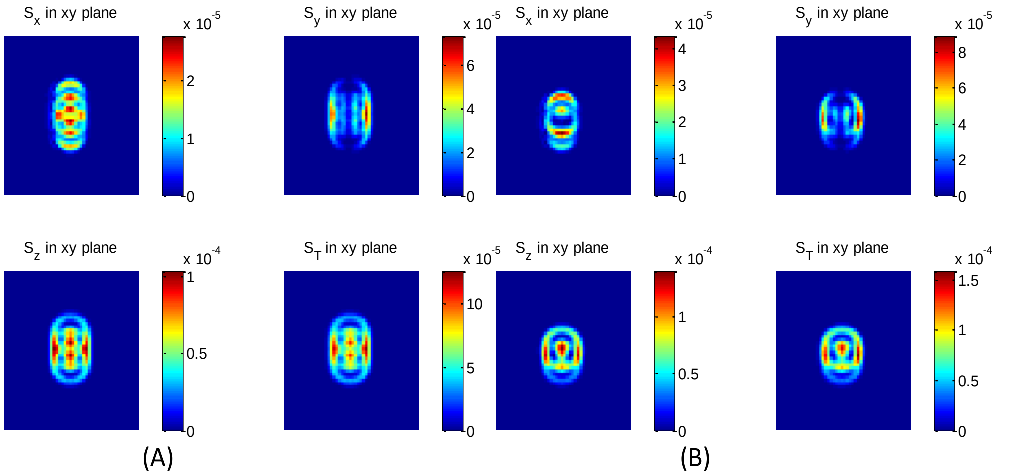

Figure 7. Images in the *z* plane which demonstrate the *Sx, Sy, Sz* and S_T. Three sources located in three locations: four cells apart (A) and source 1 placed overlapped with middle (B).

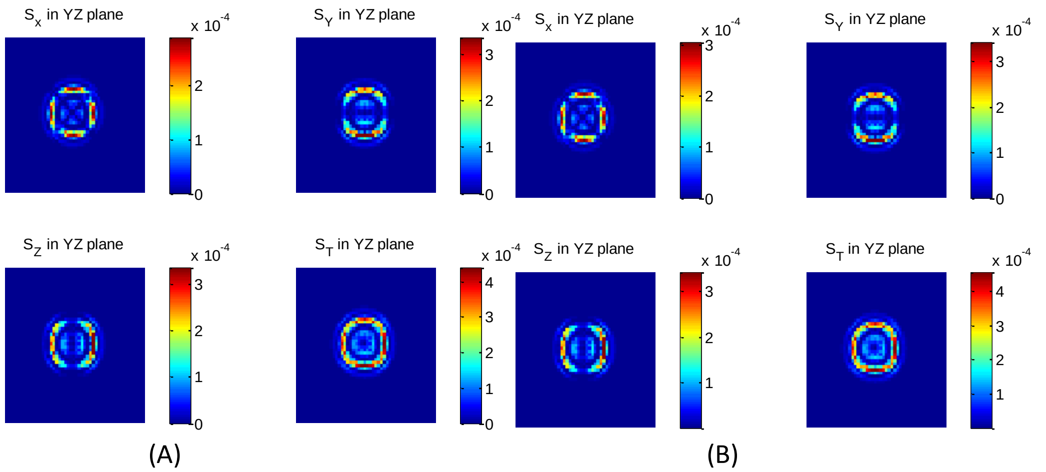

Figure 8. Images in the *x* plane which demonstrate the *Sx*, *Sy*, *Sz* and S_T. Three sources located in three locations: four cells apart (A) and source 1 placed overlapped with middle (B).

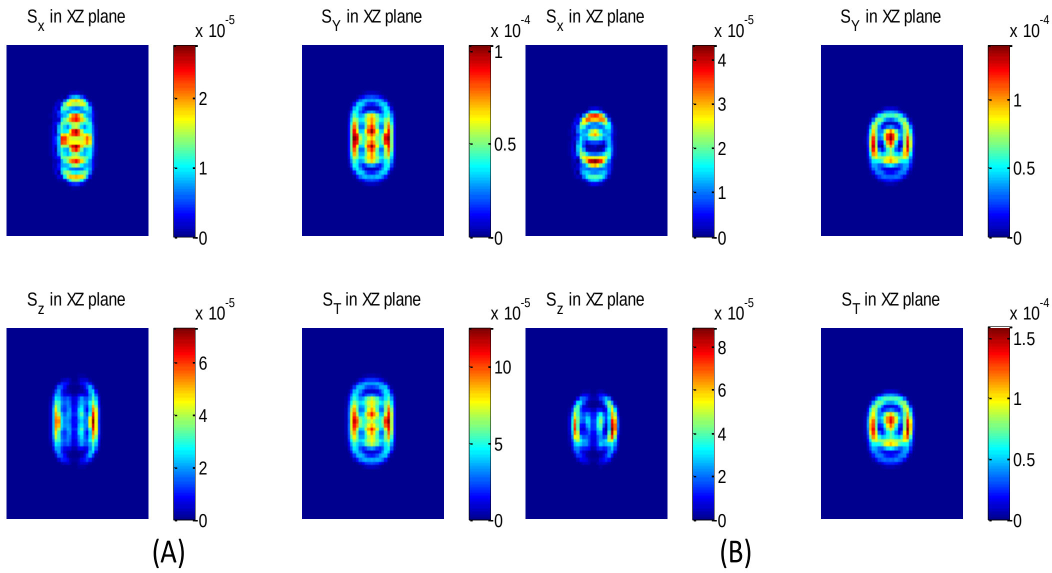

Figure 9. Images in the *y* plane which demonstrate the *Sx*, *Sy*, *Sz* and S_T. Three sources located in three locations: four cells apart (A) and source 1 placed overlapped with middle (B).

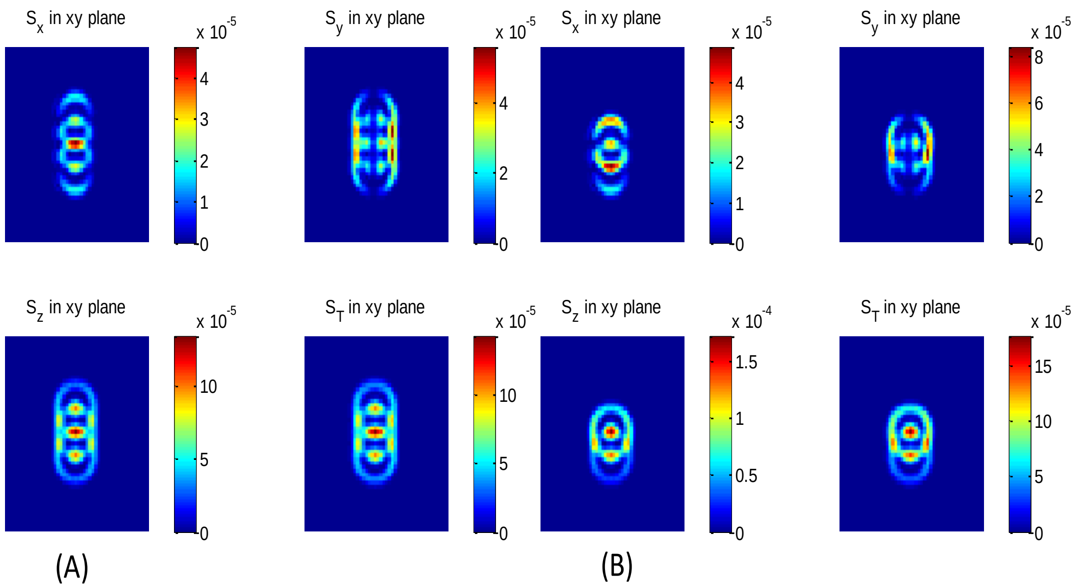

Figure 10. Images in the z plane which demonstrate the *Sx*, *Sy*, *Sz* and S_T. Three sources located in three locations: six cells apart (A) and source 1 placed overlapped with middle (B).

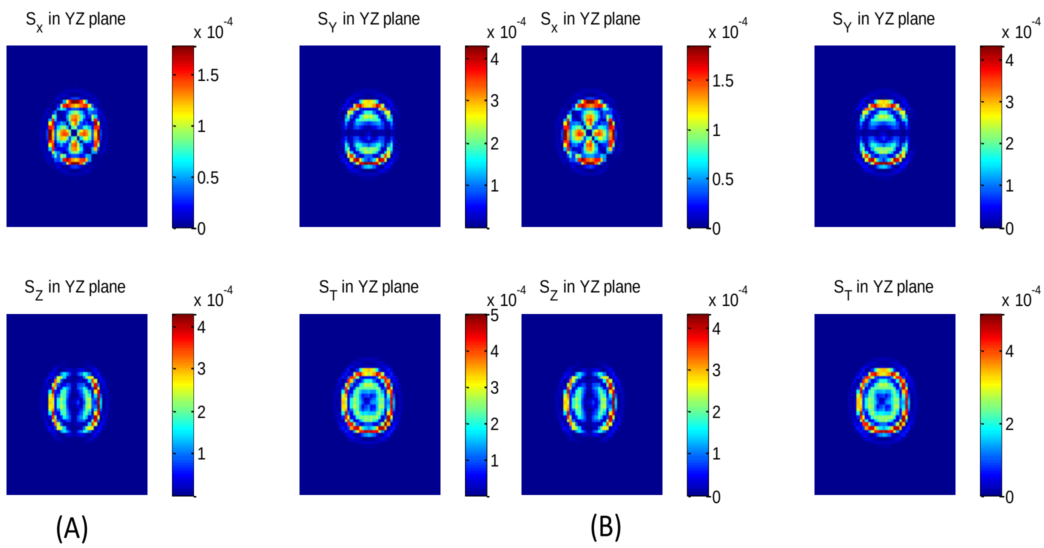

Figure 11. Images in the *x* plane which demonstrate the *Sx*, *Sy*, *Sz* and S_T. Three sources located in three locations: six cells apart (A) and source 1 placed overlapped with middle (B).

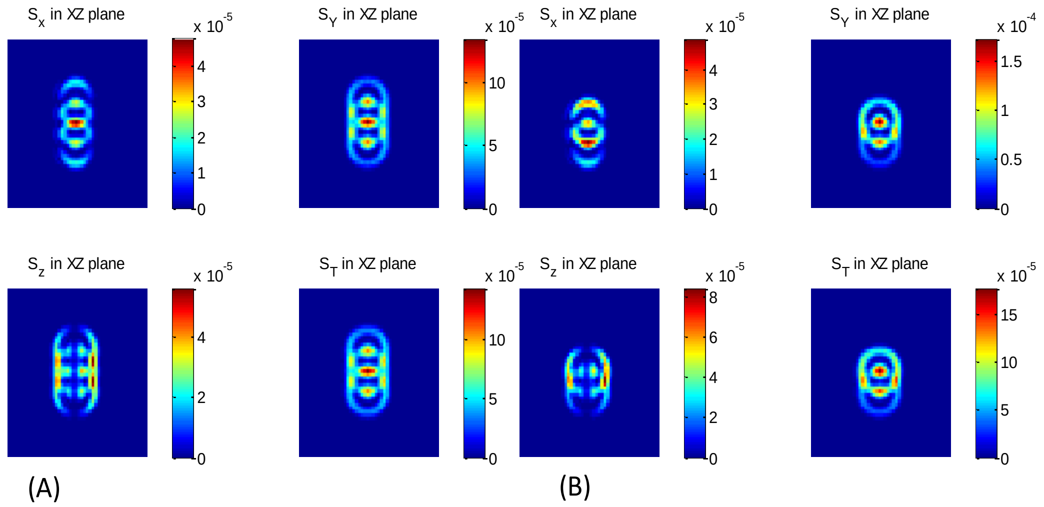

Figure 12. Images in the *y* plane which demonstrate the *Sx*, *Sy*, *Sz* and S_T. Three sources located in three locations: six cells apart (A) and source 1 placed overlapped with middle (B).

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