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Estimation of Forcing Power-Up on Doubling of CO_2 Concentration and Estimation of Temperature Rise in the Earth's Atmosphere By Three Models

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Abstract

We analyzed three different models to estimate the forcing power resulting from doubling CO_2 concentration in the air and its effect on temperature increase in the atmosphere and in particular to different regions on Earth. One of these models follow closely the one presented in an earlier study by Wilson. The models vary in complexity and can be applied to any greenhouse gas other than CO_2.

Keywords: Forcing power; CO_2 concentration; Temperature rise.

المستخلص

قمنا بتحليل ثلاثة نماذج مختلفة لتقدير قدرة التأثير الناتجة عن مضاعفة تركيز غاز ثاني أكسيد الكربون CO_2 في الهواء، وتأثيره على زيادة درجة حرارة الغلاف الجوي على مناطق مختلفة من سطح الأرض. أحد هذه النماذج ينطبق تماماً مع دراسة سابقة أجراها ويلسون. هذه النماذج تختلف من حيث التعقيد ويمكن تطبيقها على أي غاز من غازات الاحتباس الحراري غير ثاني أكسيد الكربون.

Introduction

The Earth receives approximately all the energy of the climate system from the sun in the form of electromagnetic radiation [1]. The amount of the solar power per unit area on Earth is the total solar irradiance (TSI) formally called the solar constant (I_0) [1, 2], (Fig. 1).

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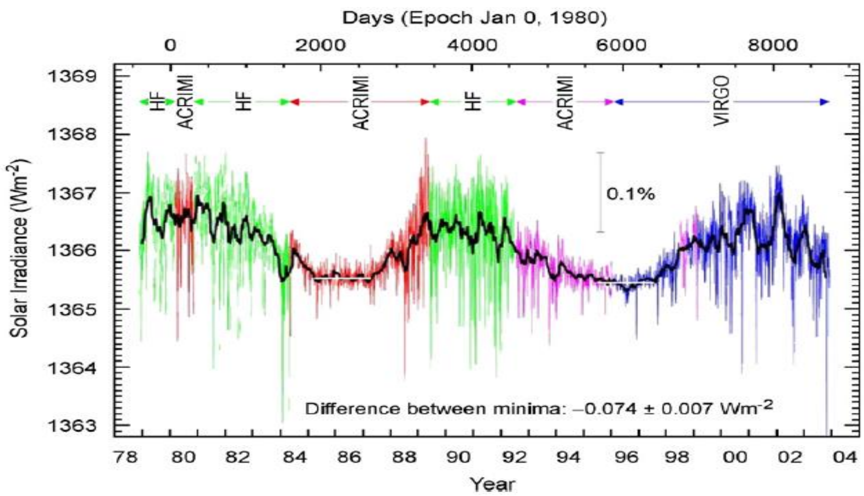

Fig. 1**.** Measurement of the total solar irradiance outside the atmosphere [3].

We presume that the Earth is in thermal radiative equilibrium, so the incoming absorbed and outgoing energy emitted must balance [2], as depicted in Figs. 2 and 3. The total absorbed radiation in an effective normal area of πR_e^2 with 30 % being reflected by the Earth albedo (α) and the rest of the energy is remitted as Infrared radiation IR from the Earth with an effective area of $4\pi R_e^2$.

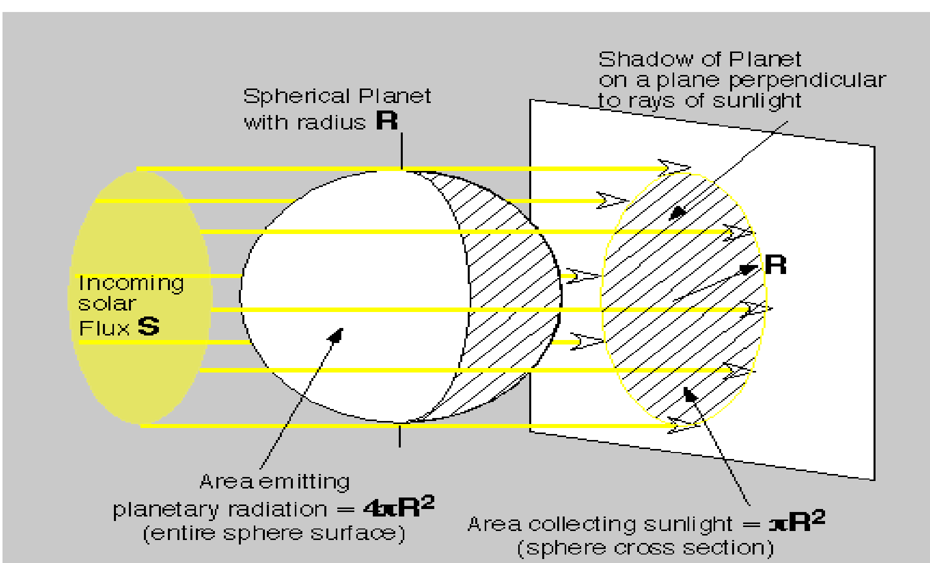

Fig. 2. Solar flux intercepted by the Earth is contained the cross sectional area πR_e^2, while the power is emitted in the all directions from a total surface area of the Earth $4\pi R_e^2$ [4].

So the basic energy balance is satisfied by
Absorbed solar radiation = emitted terrestrial radiation
Hence without atmosphere or one with emissivity ($\varepsilon = 0$), we get (Fig. 3)

$$I_0\ (1-\alpha)\ \pi R_e^2 = 4\pi R_e^2\ \sigma\ T_o^4 \quad (1)$$

leading to

$$T_o = \sqrt[\frac{1}{4}]{\frac{(1-\alpha)\ I_0}{4\ \sigma}} \quad (2)$$

where α is the average albedo of the Earth (Fig. 4) [5], σ is the Stefan- Boltzman constant ($\sigma = 5.67 \times 10^{-8}\ W/\ m^2K^4$) and T_o is the effective emission temperature of the Earth, and equal to 255 K (-18 ^{0}C for $\alpha = 0.3\ and\ I_0 \cong 1368\ W/\ m^2$). Note that we can say T_o is not the real temperature of the Earth surface but an effective one, resulting from averaging over the entire Earth surface.

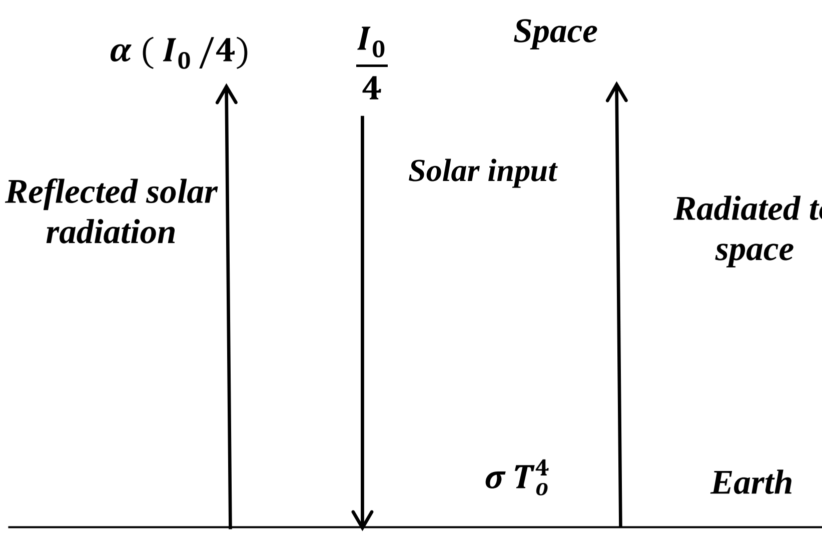

Fig. 3. The simple balance of energy without atmosphere (zero layer) on totally transparent atmosphere.

Most of the solar radiation is moved through the atmosphere of the Earth and absorbed or reflected there. In additional, the Earth is approximated a black body, so it emits radiation to the atmosphere of the Earth and back into space. The atmosphere, however, contains many gases some of which are "greenhouse gases" that absorbs some of the IR and emit radiation downwards and upwards, causing an increase in the temperature of the Earth's surface [6,7]. Fig. 5 shows the important greenhouse gases absorption spectrum [8].

Radiation Balance

From Fig. 5, we can deduce that the atmosphere is not totally transparent to infrared radiation IR. The greenhouse gases reduce the total intensity that is emitted out to space. The estimation of the contribution of greenhouse gases to air temperature increase is an active current research and a very complicated matter. Complicated programs are used to estimate how much radiation trapped in the atmosphere that contributes to a global warming trend. Atmospheric science is composed of many disciplines of science and contains competing effects that cancel each other. The climate cycle is also not very well understood. It pays however to study simple physical interactions and give reasonable simple models disciplines them.

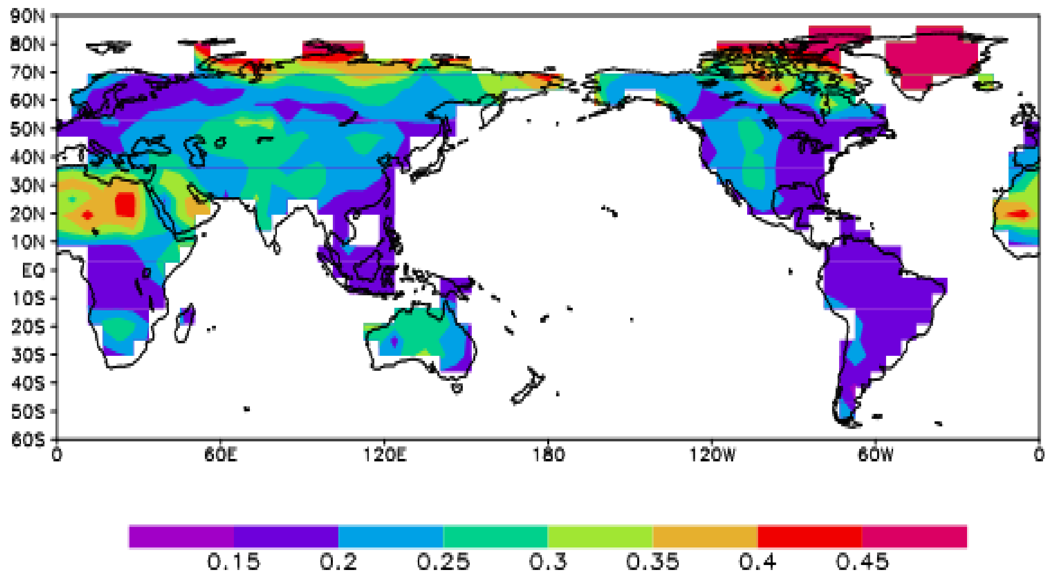

Figure 4. Albedo for the surface of the Earth.

Wilson [9] has introduced a simple model to describe the amount of IR radiation trapped into the lower atmospheric layer. He also encouraged others to improve or simplify his model. The approach in this paper is to do that by first simplifying Wilson's model. Second reduce and relax some of the approximations and compare all of the three versions of the model. But first let us elaborate on radiative balance of the Earth's atmosphere by introducing more non transparent layers.

Consider two layers of the atmosphere and the surface of the Earth [10] (Fig. 6). The net balanced flux at the surface of the Earth B_s is given by

$$B_s = I + \varepsilon_1 B_1 + (1 - \varepsilon_1)\varepsilon_2 B_2 \qquad (3)$$

Where $B_i = \sigma\, T_i^4$, $I = (1-\alpha)\frac{I_0}{4}$ and ε_i is the emissivity of the atmosphere layer. ($\varepsilon = 1$ totally opaque and $\varepsilon = 0$ totally transparent).
At the top of the first layer, the radiative balance is

$$\varepsilon_1 B_1 + (1-\varepsilon_1)B_S = \varepsilon_2 B_2 + I \tag{4}$$

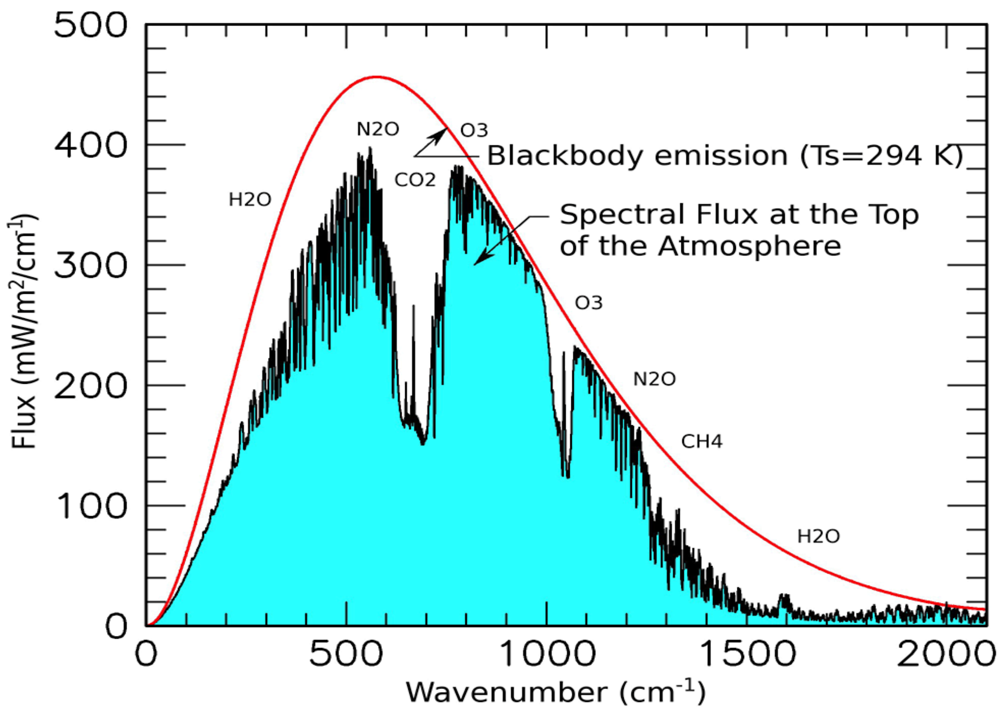

Fig. 5. Spectrum of solar radiation at the top of the atmosphere with energy absorbed by water vapour, carbon dioxide, ozone and other greenhouse gases.

At the top of the second layer

$$\varepsilon_2 B_2 + (1-\varepsilon_2)[(1-\varepsilon_1)B_s + \varepsilon_1 B_1] = I \tag{5}$$

These balanced radiation equations (3, 4 and 5) have the following solutions (using simple algebra)

$$B_2 = I\left[\frac{1}{2-\varepsilon_2}\right] \tag{6}$$

$$B_1 = I\left[\frac{1}{2-\varepsilon_1} + \frac{\varepsilon_2}{2-\varepsilon_2}\right] \tag{7}$$

$$B_s = I\left[1 + \frac{\varepsilon_1}{2-\varepsilon_1} + \frac{\varepsilon_2}{2-\varepsilon_2}\right] \tag{8}$$

It can also be shown that

$$B_s = 2\,B_1 - 2\,B_2 \tag{9}$$

This last Equation will be used in our work to estimate the temperature rise of the atmosphere.

Note that ε_i is a weighted average over frequency and $B_i = \sigma\,T_i^4$ is also the frequency integrated Blank function and according to Kirchhoff's law emission and absorption coefficients are equal.

Wilson (op. cit.) used one effective temperature for the whole Earth, because most of the area for the Earth surface is seas and oceans; about 71% (thermal bath), while the percentage of the area that have lands is approximately 29% [11]. Therefore Wilson took the temperature of the Earth's surface to be 288K. In fact, there are big differences in temperature of the surface of the Earth from one region to another, resulting from the variations with latitude and topography of the Earth.

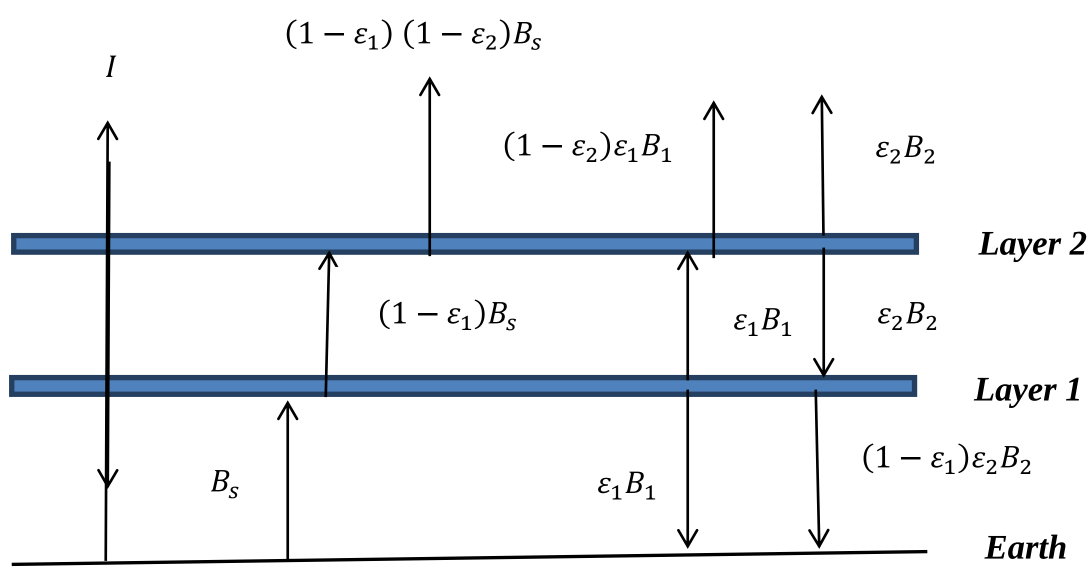

Fig. 6. Two layers of the atmosphere absorbing some of the IR radiation impacted on them.

Crude Scattering Atmosphere

This model is based on elastic scattering of IR photons from the Earth surface into the outer space through many vertical atmospheric layers each of which has a mean free path l thickness. Oblique and horizontal movement is ignored as it has the effect of reducing the layer thickness. This effect is corrected for by considering the spectral IR radiation that is directed vertically from a horizontal surface. The photon has 50-50 probability of going either up or down at the end of each layer. The probability that the photon returns to the Earth surface is known as "classical ruin problem" [12] and is given by $P_r = 1 - \frac{1}{N}$, where N is the number of steps or layers needed to cross the whole atmosphere and into space. For isothermal static atmosphere the air density profile at height Z is an exponential and is given by

$$n(Z) = n_0 \, e^{-Z/L} \qquad (10)$$

Where $\mathrm{n_0}$ is the density at the surface, $\mathrm{L} = \frac{\mathrm{k\,T_0}}{\mathrm{m\,g}}$ ~8 Km, m is the molecular mass. For air, $\mathrm{m} \sim 29$ u and, $\mathrm{T_0} = 273$ K. CO_2 mixes very well into the atmosphere hence it has the same density profile as air. Because of Kirchhof Law the coefficient of absorption is equal to that of emission then it's a good approximation to use the absorption cross section for CO_2 molecules as a scattering cross section and is given by the excellent approximation [9]

$$\sigma_\nu = \sigma_0 \, e^{-\,r_\pm\,|\nu - \nu_0|} \qquad (11)$$

with $\nu_0 = 667.5\ \mathrm{cm^{-1}}$, $\sigma_0 = 3.71 \times 10^{-23}\ \mathrm{m^2}$, $\mathrm{r_+} = 0.086$ cm $(\nu > \nu_0)$ and $\mathrm{r_-} = 0.092$ cm $(\nu < \nu_0)$.

The thickness of each layer then be comes

$$l_\nu(Z) = \frac{1}{\sigma_\nu \, n(Z)} = \frac{1}{\sigma_\nu \, n_0} \, e^{+Z/L} \qquad (12)$$

So the number of layers need to escape to space would be

$$N_\nu = n_0 \, \sigma_\nu \, L \qquad (13)$$

Where, $\mathrm{n_0}$ is the number of CO_2 molecules near the Earth surface. Since the concentration of CO_2 is about 390 ppm if near the surface of the Earth has $\mathrm{T} \sim 288$ K and pressure ~ 1atm, then, $\mathrm{n_0} = 9.91 \times 10^{21}\ \mathrm{molecules/m^3}$.

The IR radiation radiated by unit area by unit frequency by the Earth's surface (Black body) is given by Planck's formula (perpendicular to a horizontal surface)

$$B_\nu(T) = \frac{2\,\pi\,h\,\nu^3}{c^2} \, \frac{1}{e^{h\nu/kT} - 1} \qquad (14)$$

where h is Planck's constant, k is Boltzmann constant and T is absolute temperature.
Hence the radiation from the surface that is able to escape (I_ν^+) would be

$$I_\nu^+ = (1 - P_r) B_\nu(T) \qquad (15)$$

$$I_\nu^+ = \frac{B_\nu(T)}{N_\nu} \quad , N_\nu \geq 1 \tag{16}$$

The condition $N_\nu \geq 1$ is because $P_r \geq 0$.

In this work we make further improvement to this approximation by arguing that near the tropopause air will emit at temperature of $T_t = 217\,K$ as a black body and simple extrapolation can be made to estimate the total escaped radiation [13].

$$I_\nu^+ = x_\nu\, B_\nu(T) + (1 - x_\nu)\, B_\nu(T_t) \tag{17}$$

where $x_\nu = \frac{1}{N_\nu}$, $N_\nu \geq 1$. Fig. 7 shows the outgoing radiation spectrum for surface temperature of T=288 K. The trapped radiation is given by

$$\Delta I_\nu = B_\nu(T) - I_\nu^+ = (1 - x_\nu)\, [B_\nu(T) - B_\nu(T_t)] \tag{18}$$

The calculated traped IR due to CO_2 can be seen in Fig. 8 in the frequency range of $500\ \mathrm{cm}^{-1} < \nu < 800\ \mathrm{cm}^{-1}$. This is a very simple model based on our modification of Wilson's first model.

Wilson Model

Wilson used Schwarzschild's Equation [14] combined with Kirchoff's law [15] to compute the escaping IR intensity I_ν^+ at a particular frequency, ν, i.e

$$\frac{dI_\nu^+}{dZ} = -\alpha_\nu\, [I_\nu^+ - B_\nu(T(Z))] \tag{19}$$

where α_ν is the reciprocal of the absorption length, it can be written as $\alpha_\nu = \frac{1}{l_\nu} = \sigma_\nu\, n(Z) = n_0\, \sigma_\nu\, e^{-Z/L}$ (Isothermal atmosphere) with a change of variables

$$\xi = \frac{1}{N_\nu} \int_0^Z \alpha_\nu(\acute{Z})\, d\acute{Z} = (1 - e^{-Z/L}) \tag{20}$$

Then Eq. (19) becomes

$$\frac{dI_\nu^+}{d\xi} = -N_\nu\, [I_\nu^+ - B_\nu(T(\xi))] \tag{21}$$

So the formal solution of Eq. (21) is

$$I_\nu^+(\xi) = I_\nu^+(0)\, e^{-N_\nu\, \xi} + N_\nu \int_0^\xi e^{-N_\nu\,(\xi - \acute{\xi})} B_\nu\left(T(\acute{\xi})\right) d\acute{\xi} \tag{22}$$

Wilson gave the following approximate extrapolated solution for I_ν^+

$$I_\nu^+(\xi = 1) \cong B_\nu(T_0)\, e^{-N_\nu \acute{\xi}} + \left(1 - e^{-N_\nu \acute{\xi}}\right) B_\nu(T(\xi_1)) \qquad (23)$$

We can write the blocked radiation (Fig. 10) by

$$\Delta I_\nu = B_\nu(T) - I_\nu^+ = \left(1 - e^{-N_\nu \acute{\xi}}\right)[B_\nu(T_0) - B_\nu(T(\xi_1))] \qquad (24)$$

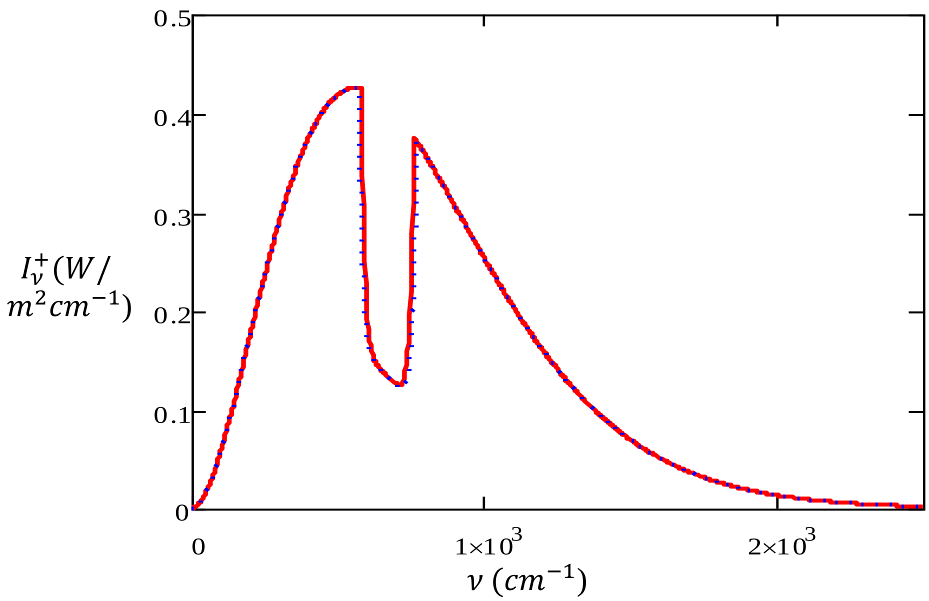

Fig. 7. Radiation intensity at the top of the atmosphere due to the CO_2 absorption for the concentration of 390 ppm (solid line) and 780 ppm (dotted line).

Equation 24 was used by Wilson and known as Wilson Second Model. This approximate model takes into account the lapse rate of temperature change in the atmosphere but does not include corrections of air density changes for non-isothermal atmosphere as well as oblique path length connections and cross section narrowing due to temperature decrees.

Full Optical Depth Solution

Equation (22) is slightly hard to solve numerically but can be transformed to much simpler version that is easier for numerical integration and hence no need for the use of extrapolation method.

Let $\quad q = e^{-N_\nu(\xi - \acute{\xi})} \qquad (25)$

then $\quad dq = N_\nu\, e^{-N_\nu(\xi - \acute{\xi})}\, d\acute{\xi} \qquad (26)$

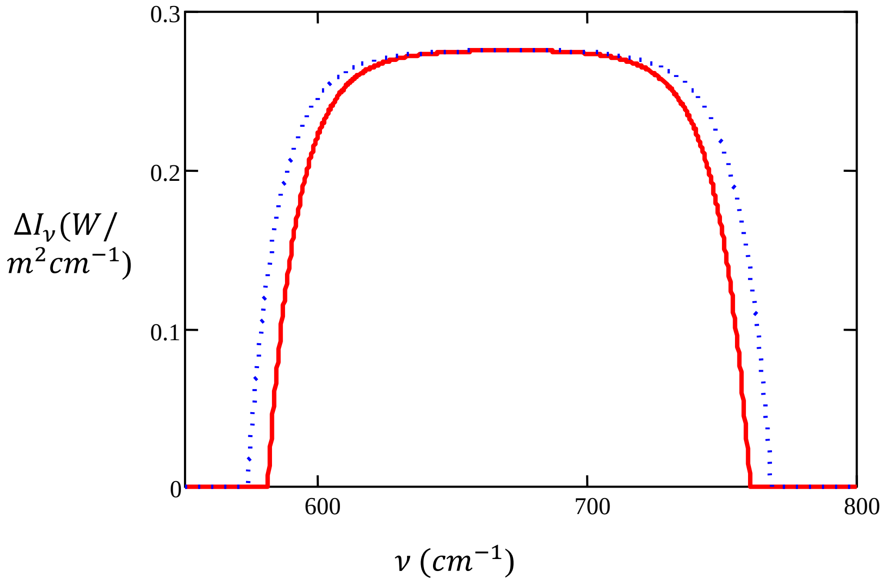

Fig. 8. The blocked intensity from CO_2 concentration; solid line $c = 390 \ ppm$ and dotted line $780 \ ppm$.

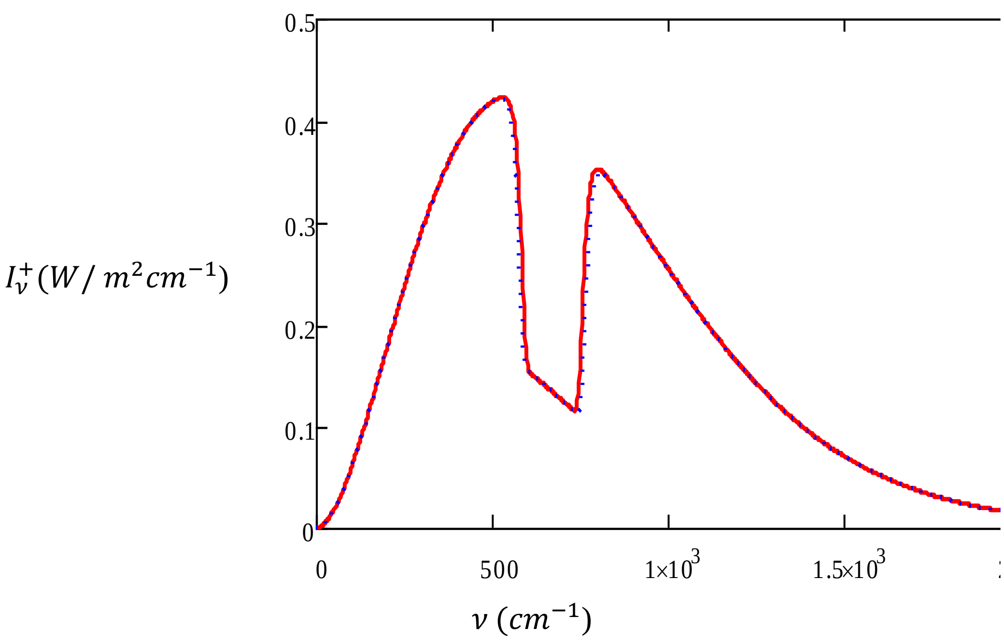

Fig. 9. Intensity of radiation at the top of the atmosphere; solid line current CO_2 concentration and the effect of doubling CO_2 concentration by Wilson model.

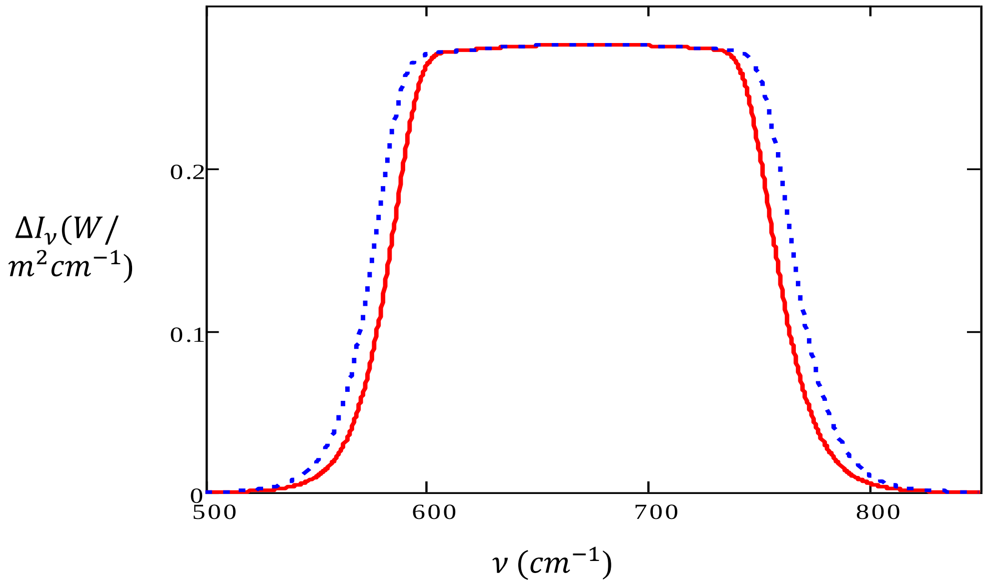

Fig. 10. The blocked IR intensity calculated from Equation (24) for Wilson model.

for $\quad \acute{\xi} = 0 \qquad q = q_m = e^{-N_\nu \xi}$ (27)

for $\quad \acute{\xi} = \xi \qquad q = 1$

Hence Equation (22) becomes

$$I_\nu^+(q_m) = I_\nu^+(q=1)\, q_m + \int_{q_m}^{1} B_\nu\big(T(q)\big) dq \tag{28}$$

Note that $I_\nu^+(q=1) = B_\nu(T_s)$, where T_s is surface temperature.

Fig. 11 gives the full solution of Eq. (22 or 28) (numerical integration). It describes the effect of optical depth on the intensity that is emitted from the atmospheric layers up to optical depth of ($\xi = 0.0, 0.3, 0.6\ and\ 0.75$)

The net trapped radiation is given by:

$$\Delta I_\nu = B_\nu(T) - I_\nu^+(0)\, e^{-N_\nu \xi} + N_\nu \int_0^{\xi} e^{-N_\nu(\xi - \acute{\xi})} B_\nu\left(T\big(\acute{\xi}\big)\right) d\acute{\xi} \tag{29}$$

$$\Delta I_\nu = B_\nu(T)[1 - q_m] + \int_{q_m}^{1} B_\nu\big(T(q)\big) dq \tag{30}$$

where $q_m = e^{-N_\nu \xi}$.

Fig. 12 shows the change in the trapped IR radiation with different heights of the atmosphere ($Z = 2Km, 4Km, 6Km, 8Km\ and\ 11Km$, by using Eq. (20) to get $\xi = 0.221, 0.393, 0.528, 0.632\ and\ 0.75$ respectively.

Forcing Power

We can calculate the change in the trapped IR (forcing power) resulting from doubling in the CO_2 concentration for three models by the following Equation

$$\delta(\Delta I_\nu) = \int_0^\infty (\, \Delta I_\nu \,(780ppm) - \Delta I_\nu \,(390ppm))\, d\nu \quad (31)$$

where 780 ppm is the double of the CO_2 current concentration.

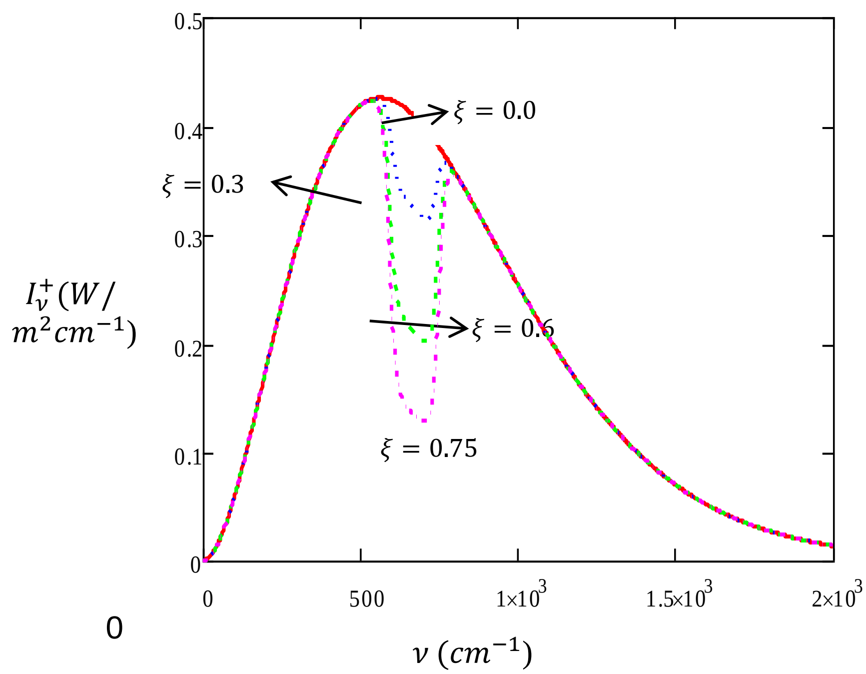

Figure 11. Emission intensity through the atmosphere as a fraction of ξ.

We use Eqs. (18), (23) and (30) to get the dependence of forcing power on surface temperature which are shown in Table 1. Fig. 13 shows the relationship between temperature and the change in the blocked intensity resulting from the change in concentration of CO_2 for the three models. As can be seen all these three models result in very close results and differ only by few parts in a thousand.

This is a very important result. It shows that varying the model for estimating the forcing power is not very significant in case of CO_2, so for future work one can depend on simple models to estimate the forcing power due to CO_2 gas.

Furthermore, it shows that all three models predict nearly linear temperature behavior for the forcing power making it easier to predict by using fewer temperature calculations. Another important point here is that the forcing power is mainly linear and different from what Wilson proposed i. e., $\delta(\Delta I)\ \propto c\, T_o^4$ for his calculation of temperature rise ($\delta\, T$).

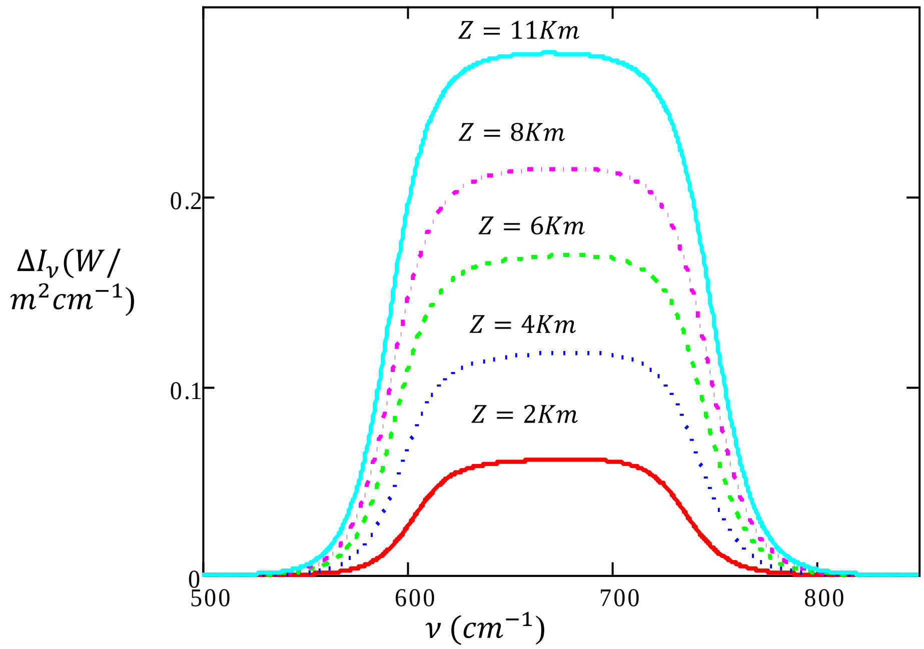

Fig. 12. Net trapped IR of full optical depth model resulting from Equation (25 or 26).

Table 1. The forcing power as a function of surface temperature for scattering, Wilson and full optical models when n_0 *and* L are constant (isothermal atmosphere).

T_s (K)	$\delta(\Delta I_\nu)$ for Crude Scattering (W/m^2)	$\delta(\Delta I_\nu)$ for Wilson Model (W/m^2)	$\delta(\Delta I_\nu)$ for Full Optical (W/m^2)
220	0.127	0.128	0.128
230	0.582	0.586	0.588
240	1.086	1.092	1.096
250	1.639	1.647	1.653
260	2.238	2.25	2.257
270	2.884	2.898	2.908
280	3.576	3.593	3.605
290	4.311	4.331	4.345
300	5.089	5.111	5.128
310	5.907	5.933	5.952
320	6.766	6.794	6.815
330	7.662	7.693	7.716

There is a quadratic regression between the amount of the forcing power and temperature that is given by next Equation:

$$\delta(\Delta I_\nu) = a\,(T_s - 217) + b\,(T_s - 217)^2 \qquad (32)$$

Where the units of a are $(W/m^2\,K)$ and b are $(W/m^2\,K^2)$.

$$\delta(\Delta I_\nu)^{crude\ scattering} = 0.0422(T_s - 217) + 2.3236 \times 10^{-4}\,(T_s - 217)^2 \quad (33)$$
$$\delta(\Delta I_\nu)^{Wilson\ app.} = 0.0423(T_s - 217) + 2.3292 \times 10^{-4}\,(T_s - 217)^2 \quad (34)$$
$$\delta(\Delta I_\nu)^{full\ optical} = 0.0419(T_s - 217) + 2.3198 \times 10^{-4}\,(T_s - 217)^2 \quad (35)$$

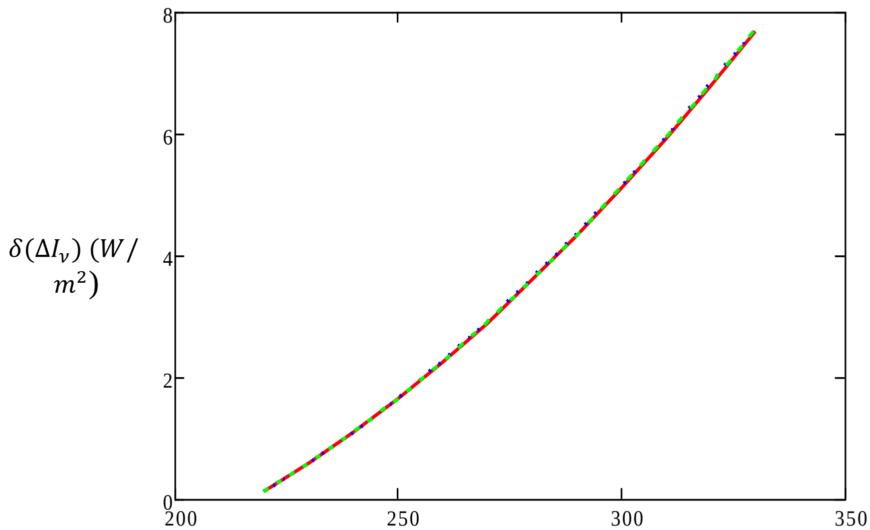

Fig. 13. Relationship between surface temperature and forcing power for the three models.

Estimating the Temperature Rise

Wilson estimated that the effect of the greenhouse reduces the heat escaping the Earth to the outer space hence increasing the Earth surface temperature. His balance Equation is then

$$(1 - x)\sigma\, T_s^4 = \frac{(1-\alpha)}{4}\, I_0 \quad (36)$$

For $T \sim 288\ K\ and\ x = 0.39$. The increase in T due to increase of x is then given by

$$\delta\, T^W = \frac{T_s^5}{4\, T_0^4}\, \delta\, x \quad (37)$$

Wilson assumed the traped Infrared radiation is ΔI^+ and $\delta\, x = \frac{+\,\delta(\Delta\, I^+)}{\sigma\, T_s^4}$ so Eq. (37) leads to

$$\delta\, T^W = \frac{T_s\, \delta(\Delta\, I^+)}{4\, \sigma\, T_0^4} \quad (38)$$

In this work we argue that such assumption violates radiation balance and we will arrive at different estimate.

For one layer system of thermodynamic atmosphere we get the following Eqs.

$$B_s = I + \varepsilon_1\, B_1 \quad (39)$$
$$(1 - \varepsilon_1)B_s + \varepsilon_1\, B_1 = I \quad (40)$$

Therefore,

$$B_1 = I\left(\frac{1}{2-\varepsilon_1}\right) \tag{41}$$

$$B_s = 2\,B_1 \tag{42}$$

We note here

$$B_s = I\left(\frac{2}{2-\varepsilon_1}\right) \tag{43}$$

or

$$\left(1-\frac{\varepsilon_1}{2}\right)B_s = I \tag{44}$$

Which is the same as a Equation (36) for Wilson but with $= \frac{\varepsilon_1}{2}$.

For these balanced equations $\delta B_s = 2\,\delta B_1$ and we know all traped energy will end up in the atmosphere layer for simplicity then $\delta B_1 = \delta(\Delta I^{+})$.

Hence

$$\delta B_s = 2\,\delta(\Delta I^{+}) \tag{45}$$

$$4\,\sigma\,T_s^3\,\delta T_s = 2\,\delta(\Delta I^{+}) \tag{46}$$

Therefore,

$$\delta T_s = \frac{\delta(\Delta I^{+})}{2\,\sigma\,T_s^3} \tag{47}$$

This is a more reasonable estimate of temperature increase due to any kind of greenhouse gases that satisfies the radiation balance.

$$\frac{\delta\,T_s}{\delta\,T^W} = \frac{\delta\,(\Delta\,I^{+})/2\,\sigma\,T_s^3}{T_s\delta\,(\Delta\,I^{+})/4\,\sigma\,T_0^4} = 2\left(\frac{T_0}{T_s}\right)^4 \tag{48}$$

This Equation clearly indicates that our estimate of temperature rise that satisfies the radiation balance is larger than the one estimated by Wilson up to $T_s \sim 303$ K.

We can also estimate the minimum temperature rise by setting $\delta(\Delta\,I^{+}) = \delta(\varepsilon\,B_1)$ to get

$$\delta B_s = \delta(\Delta I^{+}) \tag{49}$$

$$\delta T^m = \frac{\delta(\Delta I^{+})}{4\,\sigma\,T_s^3} \tag{50}$$

Which is half δT_s. Table 2 shows the different estimates of the change in T rising by a doubling of the concentration of CO_2. Fig. 14 shows the comparison between $\delta\,T^W, \delta T_s\; and\; \delta T^m$ for the full optical depth model.

We know that the Earth has many latitude regions that take into acount geological topology of the Earth with different variations in average temperatures and the change over the whole year. Nonetheless, the average Earth surface is temperature14^{o}C, while the in the deserts near the equator and in the hottest zones is about 57.7 ^{o}C. There is also different temperatures in Antarctica it can get below -89 ^{o}C. So in our wok, we calculate the forcing power of CO_2 for the different temperatures of the Earth's surface, it start from 220 K to 330 K.

Because $\delta(\Delta T)$ has a weak quadratic dependence on T it is expected that $\delta(\Delta\,I)$ will be evaluated at the average temperature of specific region on the Earth.

Table 2. Estimation of temperature rise by δT_s when n_0 and L are constant (isothermal atmosphere).

$T_s\,(K)$	δT_s for Crude Scattering	δT_s for Wilson Model	δT_s for Full Optical
220	0.105	0.106	0.106
230	0.422	0.424	0.426
240	0.693	0.697	0.699
250	0.925	0.93	0.933
260	1.123	1.129	1.133
270	1.292	1.299	1.303
280	1.436	1.443	1.448
290	1.559	1.566	1.571
300	1.662	1.669	1.675
310	1.749	1.756	1.762
320	1.821	1.828	1.834
330	1.88	1.888	1.893

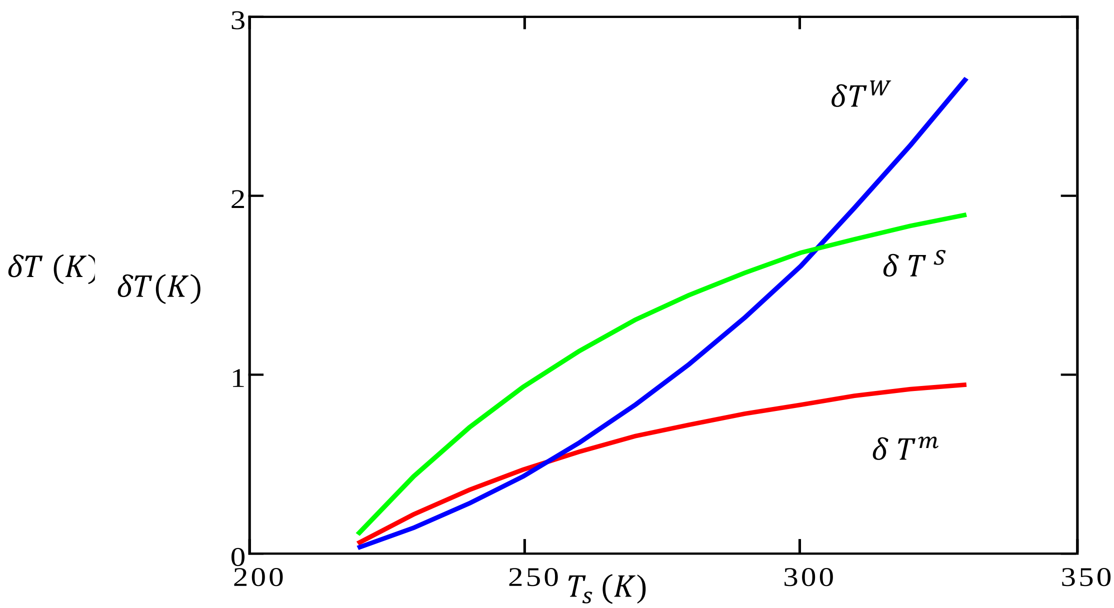

Fig. 14. Relationship between temperature rise δT by the three types of δT estimates.

$$\overline{\delta(\Delta\, I)} = a\left(\overline{T} - 217\right) + b\,(\overline{T} - 217)^2 + \tfrac{b}{3}\,(\Delta T)^2 \tag{51}$$

$$\overline{\delta(\Delta\, I)} = \delta\big(\Delta\, I(\overline{T})\big) + \tfrac{b}{3}\,(\Delta T)^2 \tag{52}$$

Where ΔT gives the variation of temperature around its average value $\overline{T}$ in the interval $(\overline{T} - \Delta\,, \overline{T} +\ \Delta)$. We get

$$\delta T = \frac{\delta(\Delta I(\overline{T}))}{2\,\sigma\,\overline{T^3}} \tag{53}$$

and

$$\Delta(\delta T) = \frac{b\,(\Delta T)^2}{6\,\sigma\,\overline{T^3}} \tag{54}$$

In Table 3 different bands of width of 20° latitude regions on Earth are shown with their respective average temperatures and the expected temperature rise for each region using the full optical model and the balanced radiation budget [16]. Of course in these estimates transfer of thermal energy from a region to another by clouds and winds or sea currents are ignored.

Table 3. Temperature rise and the change of temperature rise using Eq. (53) and Eq. (54).

Latitude (°)	$\overline{T}$ **(°C)**	Δ **T (C°)**	δT **(K)**	$\Delta(\delta T)$ **(K)**
80	-18	34	1.025	0.048
60	-1	41	1.317	0.058
40	14	19	1.518	0.011
20	25	6	1.638	9.443×10^{-4}
0	26	1	1.648	2.597×10^{-5}
-20	23	5	1.618	6.691×10^{-4}
-40	12	6	1.494	1.079×10^{-3}
-60	-3	11	1.287	4.267×10^{-3}
-80	-27	15	0.835	0.01

Conclusion

In this work we have shown that the increase of the trapped IR radiation due to doubling of current CO_2 concentration is almost a linear function of surface temperature with small positive quadratic term irrespective of the complexity of the model used to calculate it. The three models used in this work are very simple scattering model, Wilson second model and full numerical integration of Schwarzschild's equation. Deviations from all these three models are only few parts in a thousand as seen from Table 1. It is clear that one can use simple models to estimate the forcing power of CO_2 in particular. The nearly linear behavior of forcing power on surface temperature is also a very good feature allowing fewer numerical calculations and average temperatures can be used to estimate the forcing of CO_2 for different regions on Earth as well as at different seasons. Such simple estimates can make complex climate modelling easier.

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