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Impact Evaluation of the Omitted Observations Size on the Performance of the Goldfeld-Quandt Test

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Abstract

Goldfeld-Quandt test is one of the three most popular tests to detect problem of heteroscedasticity in regression analysis. This test was proposed by Goldfeld and Quandt in 1965. It is implemented as follow: (1) record the data set according to the values of the independent variable, which is suspected to be cause of heteroscedasticity, from lowest to highest, (2) divide the sample size, n, that was already sorted to three parts and omit the middle part with size c. Thus, we obtain two subsamples of sizes n_1 and n_2, usually $n_1 = n_2 = (n-c)/2$. If heteroscedasticity is present, then the variance of the last subsample will not be the same as the variance of the first subsample; it tends to be larger. The *F*-test for the ratio of the two variances can be used to test for the equality of variances. The ability of the Goldfeld-Quandt test to detect the heteroscedasticity problem is likely to be sensitive to the size of middle part, *c*, that should be discarded. In this work a simulation study was conducted to determine the appropriate value of c to make the Goldfeld-Quandt test more effective. The results of the simulation study confirmed that the appropriate size of the omitted values, c, should not be less than 30% of the sample size, n, in order to ensure a best performance of the Goldfeld-Quandt test.

Keywords: Heteroscedasticity; Homoscedasticity; Power of the test; Mont Carlo simulations; Regression analysis.

المستخلص

يعتبر اختبار غولدفيلد-كواندت من أكثر الاختبارات المعروفة للكشف عن مشكلة عدم تجانس التباين في تحليل الانحدار والذي اقترحه كل من Goldfeld و Quandt في سنة 1965. هذا الاختبار يتم تنفيذه كالتالي: (1) إعادة ترتيب مجموعة البيانات من أصغر قيمة إلى أكبر قيمة وفقا لقيم المتغير المستقل الذي يشتبه في أنه سبب في عدم تجانس التباين،(2) تقسيم العينة ذات الحجم n التي تم إعادة ترتيبها تصاعدياً إلى ثلاثة أجزاء وحذف الجزء الأوسط ذو الحجم c، وبالتالي نتحصل على عدد

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اثنين من العينات الفرعية بالأحجام n_1 و n_2 على التوالي، وعادة ما تكون $n_1 = n_2 = (n-c)/2$ ، (3) إذا كان عدم تجانس التباين موجود، فإن تباين العينة الفرعية الثانية لن يكون مساوي لتباين العينة الفرعية الأولى. يمكن استخدام اختبار F للنسبة بين تباينين لاختبار تساوي تباين العينتين الأولى والثانية. من المرجح أن تكون قدرة اختبار غولدفيلد–كواندت للكشف عن مشكلة عدم تجانس التباين حساسة لحجم الجزء الأوسط، c . في هذه الورقة تم إجراء دراسة محاكاة لتحديد الحجم المناسب للقيم التي يجب إهمالها من منتصف البيانات، c ، وذلك لجعل اختبار غولدفيلد–كواندت أكثر فاعلية. أكدت نتائج دراسة المحاكاة أن الحجم المناسب للقيم التي يتم اهمالها، c ، يجب أن لا تكون أقل من 30% من حجم العينة الكلي، n ، لأجل ضمان أفضل أداء لاختبار غولدفيلد–كواندت.

Introduction

One requirement of statistical inference in ordinary least squares (OLS) is that the error variances, σ^2, is the same across all the observations. This requirement is known as the homoscedasticity assumption. Under this assumption and the other set of usual assumptions, the estimators determined by OLS are best linear unbiased estimators (BLUE). Thus, the statistical inference in OLS is valid. The heteroscedasticity occurs when the error variances are not the same across observations, which means that heteroscedasticity violates the homoscedasticity assumption. It is well known that when the equality of error variance assumption is violated, the OLS may results an inefficient estimate and hence the usual tests of significance are invalid. Therefore, the heteroscedasticity problem should not be ignored and it is necessary to detect its presence. There are many tests for detecting the presence of heteroscedasticity. The Goldfeld-Quandt test is utilized if the variance of error term σ_i^2 is certainly related to one of the independent variables.

Goldfeld and Quandt (1965) suggested the following steps for null and alternative hypotheses of the form:

$H_o : \sigma_i^2$ is constant for all observations (Homoscedasticity)

$H_1 : \sigma_i^2$ is not the same for all observations (Heteroscedasticity)

Step 1. Order or rank the observations according to the values of X_i, beginning with the lowest X value.

Step 2. Divide the sample that has already been reordered into three groups, then, omit the middle group of c observations. Two groups remain, each one has $n_1 = n_2 = (n-c)/2$ observations.

Step 3. Fit separate OLS regressions to the first n_1 observations and the last n_2 observations, and obtain the respective residual sums of squares RSS_1 and RSS_2; RSS_1 representing the RSS from the regression corresponding to the smaller X_i values (the small variance group) and RSS_2 represents the larger X_i values (the large variance group).

The RSS_1 and RSS_2 have the same degrees of freedom when the two subsamples have the same number of observations. Hence,

$$n_1 = n_2 = (n-c)/2 \text{ , and } df_1 = df_2 = \frac{(n-c)}{2} - k = \left(\frac{n-c-2k}{2}\right)$$

Where k is the number of parameters to be estimated including the constant term.

Step 4. Compute the *GQ* statistic

$$F_{GQ} = \frac{(RSS_2/df_2)}{(RSS_1/df_1)} \quad (1)$$

Under the null hypothesis H_o (homoscedasticity assumption) with the errors ε_i normally distributed (usual assumption), the F_{GQ} statistic has *F* distribution with numerator and denominator degrees of freedom $(n-c-2k)/2$ (Gujarati, 2003).

Reject the null hypothesis H_o at significance level α and conclude that heteroscedasticity is present if the *p-value* of the test are less than or equal to α .

Literature Review

Goldfeld and Quantd (1965) proposed using the test for heteroscedasticity after the omission of some number of middle observations. The purpose of omitting some of middle observations is to make the difference between the two error variances clearer. If few observations are omitted the test may fail to detect the heteroscedasticity problem. But, omitting too many observations diminishes the size of subsamples. Therefore, lower degrees of freedom in the estimation with each subsample, and this tends to lower the power of the test. The deletion of c values with careful selection of the size of omitted observations will increase the power of the test. Goldfeld and Quantd did not specify how many observations should be removed. They gave the relative frequency of cases in which the false hypothesis is rejected for samples of dimension $n = 30$ and $n = 60$ after omitting 0, 4, 8, 12 or 16 central observations which estimated the power of the test. They obtained the largest frequency for $n = 30$ and $n = 60$ after the omission, respectively, of 8 and 16 central observations (equal to 26.1%). Buse (1984), analysed the problem for $n = 20,40,80$ removing 20% of central observations. The same dimension of the removed set of observations was used by Dufour et al. (2004) for $n = 50$ and $n = 100$. Maddala (1992) suggests the removal of central observations to increase the power of the test, but he does not answer the question of how many observations to remove.

Judge et al. (1982) suggest that $c = 4$ if $n = 30$ and $c = 10$ if n is about 60, which have been found satisfactory in practice (Gujarati, 2003). Monte Carlo's experiment was based on discard of around 25% of the observations (Griffiths and Surekha, 1985; Creel, 2014).

Williams (2015), indicted that, typically, c might equal 20% of the sample size. This technically is not necessary, but experience shows that this tends to improve the power of the test. Abbott (2009) showed that c is arbitrarily chosen to equal some value between $n/6$ and $n/3$. Gau (2002), stated that the number of observations to be omitted is arbitrary and usually between one-sixth and one-third. Note that n_1 and n_2 must be greater than the number of coefficients to be estimated.

Shalabh and Kanpur (2007) stated that one difficulty in Goldfeld-Quantd test is that the choice of c is not obvious. The basic objective of ordering of observations and deletion of observations on the middle part may not reveal the heteroscedasticity effect. Since the first and last values of σ_i^2 give the maximum discretion, so deletion of smaller value may not give the proper idea of heteroscedasticity. The working choice of c is suggested as $c = n/3$.

Griffiths and Surekha (1985) used sample sizes of $n = 20$ and 50 and they set $c = 4$ and 10, respectively. Djolov (2002) omitted the middle 5 observations of $c = 50$ sample size. Harvey and Phillip (1974) suggest that no more than a third of the observations should be dropped.

Heteroscedastic Variance Structures

In this study one type of heteroscedasticity structure is considered with two different degrees. The heteroscedasticity structure specification is:

$$V(\varepsilon_i) = \sigma_i^2 = kX_i^{\gamma} . \tag{2}$$

This model has been discussed by Geary (1966), Park (1966), Lancaster (1968), Kmenta (1971), and Harvey (1976).

Simulation Data Model

The Goldfeld-Quandt test assumes that the heteroscedasticity variance depends on one of the independent variables in the model. We consider the model with a single independent variable:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i , \tag{3}$$

for $\beta_0 = 4$ and $\beta_1 = 5$, were held constant across all simulation conditions. The error terms ε_i were generated from a normal distribution with mean zero and variances given by (2). We employed this model of variance structure with two different degrees for the heteroscedasticity as:

$$V(\varepsilon_i) = \sigma_i^2 = \sigma^2 X_i , \tag{4}$$

$$V(\varepsilon_i) = \sigma_i^2 = \sigma^2 X_i^2 , \tag{5}$$

for $\sigma = 0.65$ and all of the other classical assumptions still hold.

The independent variable X is non-stochastic variable with n elements randomly selected from the numbers 1 to 20. The values of the dependent variable Y are obtained from Eq. (3).

Simulation Study

The data for this study were generated and the Goldfeld-Quandt test was conducted using MATLAB software. In order to get different values of the power of the test and then to pick the size c that has greatest power test a simulation study was run using various sizes of sample (n = 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, 210, 220 and 250). Then for each sample size, 150,000 data sets were generated, we numbered the omitted values and c as ratios of sample size (c= 0%, 1%, 2%, . . , 60%) were varied. The power of test was estimated by calculating the proportion of rejections in 150,000 replications at a 5% level of significance.

Tables 1 and 2 show the simulation results, which indicate the power of Goldfeld-Quandt test at different samples size and different omitted values as percentage of sample size.

Conclusions

The form of error variance was varied by two different construction forms, and for the proportion of the omitted values we have attempted to conduct a large-enough number of simulations for our data that show heteroscedasticity, to detect accurately how this factor (omitted values size) impacts the performance of the Goldfeld-Quandt test; i.e. to detect the heteroscedasticity problem. It was found that increasing the proportion of the omitted values results in an improvement in the performance of the Goldfeld-Quandt test.

We can say in general that the best performance of the test occurs when the size of omitted values are from $c = 30\%$ to $c = 50\%$ of the whole sample size. These results are not consistent almost with all of the value of c presented in many studies, which, surprisingly, propose that the number of omitted values should not exceed 30% of observations. From this study we can confirm that the appropriate size of the omitted values should not be less than 30% of observations in order to ensure the best performance of the test across all different samples size as well as the severity of heteroscedasticity.

Table 1. Simulated power of Goldfeld-Quandt test for error variance $=\sigma^2 X$ using $\alpha = 0.05$.

Sample Size	c 0%	20%	25%	30%	35%	40%	45%	50%	55%
20	0.3941	0.4276	**0.4296**	0.4256	0.4148	0.4065	0.3714		
30	0.5478	0.6051	0.6165	**0.6246**	0.6208	0.6140	0.6034	0.5927	
40	0.6685	0.7437	0.7519	**0.7626**	0.7618	0.7560	0.7541	0.7464	
50	0.7595	0.8311	0.8464	0.8511	0.8535	**0.8543**	0.8462	0.8485	
60	0.8237	0.8876	0.8998	0.9005	0.9056	0.9087	**0.9105**	0.9076	
70	0.8750	0.9271	0.9341	0.9432	**0.9453**	0.9452	0.9432	0.9420	0.9053
80	0.9099	0.9525	0.9573	0.9611	0.9641	0.9652	**0.9679**	0.9676	0.9668
90	0.9361	0.9670	0.9715	0.9750	0.9775	0.9779	**0.9817**	0.9809	0.9778
100	0.9552	0.9797	0.9836	0.9854	0.9871	0.9874	**0.9889**	0.9884	0.9883
110	0.9688	0.9877	0.9898	0.9916	0.9927	**0.9937**	0.9934	0.9926	0.9925
120	0.9787	0.9924	0.9938	0.9946	0.9954	0.9956	**0.9960**	0.9957	0.9951
130	0.9849	0.9954	0.9961	0.9970	0.9972	0.9977	**0.9981**	0.9978	0.9978
140	0.9897	0.9970	0.9977	0.9979	0.9985	0.9985	0.9987	**0.9988**	0.9984
150	0.9929	0.9983	0.9989	0.9990	0.9991	0.9993	0.9993	**0.9995**	0.9993
160	0.9950	0.9988	0.9991	0.9993	0.9995	0.9996	**0.9997**	0.9997	0.9995
170	0.9966	0.9992	0.9994	0.9996	0.9997	0.9997	**0.9998**	0.9998	0.9997
180	0.9977	0.9996	0.9998	0.9998	**0.9999**	0.9998	0.9998	0.9998	0.9998
190	0.9982	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
200	0.9989	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
210	0.9992	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
220	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
250	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 2. Simulated power of Goldfeld-Quandt test for error variance $= \sigma^2 X^2$ using $\alpha = 0.05$.

Sample Size	c 0%	20%	25%	30%	35%	40%	45%	50%	55%
20	0.7512	0.8025	0.8196	0.8123	**0.8197**	0.8061	0.7929	0.7709	
30	0.8953	0.9355	0.9450	0.9509	**0.9532**	0.9482	0.9456	0.9390	0.9373
40	0.9569	0.9789	0.9825	0.9845	0.9862	**0.9867**	0.9861	0.9861	0.9852
50	0.9829	0.9937	0.9950	0.9965	0.9969	0.9969	0.9972	**0.9976**	0.9970
60	0.9936	0.9981	0.9988	0.9988	0.9992	0.9991	**0.9994**	0.9992	0.9990
70	0.9976	0.9996	0.9997	0.9997	0.9998	0.9998	**0.9999**	0.9998	0.9998
80	0.9989	0.9999	0.9999	0.9999	**1.000**	**1.000**	**1.000**	**1.000**	0.9999
90	0.9997	0.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	0.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
110	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
120	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
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250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

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