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Radii of Ellipsoid Shaped Nuclei

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ABSTRACT

A new formula for evaluating the radii of deformed nuclei is proposed. By incorporating the intrinsic moment of inertia and the ground state energy $E(2)$, the formula simply predicts and reproduces the available experimentally mean square radii of deformed even–even nuclei. Calculated radii are quite close to data compared with other earlier available results.

Keywords: Rotation; deformation; inertia; quadrupole; ellipsoid.

1. Introduction

The knowledge of nuclear sizes plays an important role in understanding the structure of complex nuclei. It is also a key for studying the characteristics of nucleus and testing theoretical approaches and models.

The developments in the measurement techniques for charge radii of nuclei provide more accurate experimental results [1] which can be used to improve model parameters. The radius of nucleus can be determined from its charge density distribution [2]. Since the size of a nucleus depends mainly on its charge distribution, it is naturally proportional to the mass number *A*.

However, the conventional *A*-dependent formula, $R_0 = r_0 A^{1/3}$, is not valid for all nuclei [3], especially for those nuclei containing a significant difference between protons and neutrons numbers. Experimental data indicates that the order of magnitude of the range of nuclear forces compared with nuclear radius constant r_0 is not quite constant [4], [5]. Besides the regular *A*-dependent formula, some other approaches tending to describe nuclear size from the developed *Z*- and *N*-dependent formulae [1], [6] with relatively more reliable *N*-dependent formula [2]. On the other hand, earlier evidences indicate that a large number of nuclei can have deformed shapes [7]. These class of deformed nuclei can acquire spheroidal shapes, which likely described in terms of their semi-minor and semi-major radii.

In this work, we attempt to propose a new approach used to determine the radii of deformed even-even nuclei.

1.1. Theory and Approach

As a consequence of nuclear rotations [8], [9] a large number of nuclei can depart from spherical shapes and acquiring a spheroidal shapes, in the form of either oblate or prolate deformations [10]. Among a class of nuclei there are large number of even-even nuclei falling in the mass range between $150 < A < 180$ and $A > 250$ exhibit deformation caused by centrifugal stretching [11]. Nuclear rotations can roughly be described by the following equation [12]:

$$E_I = \frac{\hbar^2}{2\vartheta} I(I+1) \qquad (1)$$

where I and ϑ denote the nuclear spin and moment of inertia, respectively. For an axially symmetric rigid rotator with uniform mass distribution *m*, the moment of inertia is simply given by

$$\vartheta = \frac{1}{5} m(a^2 + b^2) \qquad (2)$$

where *a* and *b* are the semi-minor and semi-major axes, respectively. The nuclear deformations are considered for uniformly charged spheroid by taking the radial coordinates [13] of the surface of the nucleus

$$R = R_0[1 + \beta Y_{20}(\theta, \emptyset)] \qquad (3)$$

where the deformation parameter β is related also to the differences between the major *a* and minor *b* semi-axes as $\Delta R = a - b$, and is given by

$$\beta = \sqrt{\frac{16\pi}{45}}\,\frac{\Delta R}{R_0} \qquad (4)$$

It is assumed in the first approximation that $\beta^2 << 1$ [10], so that β is an acceptable value, thus it can be determined from the observed value of the intrinsic quadrupole moment of the nucleus [14] as

$$\beta = \frac{\sqrt{5\pi}}{3}\,\frac{Q_0}{ZR_0^{\,2}} \qquad (5)$$

where *Z* being the atomic number. It is well known that the intrinsic quadrupole moment of evenly charged ellipsoid can be described by the following equation [15]:

$$Q_0 = \frac{2}{5} Z(a^2 - b^2) \qquad (6)$$

The moment of inertia given by Eq. (2) is assumed to represent an ellipsoid of revolution of deformed nucleus, is in common with the moment of inertia represented by Eq. (1). Thus, equating these two equations for the moment of inertia ϑ, obtaining

$$\frac{1}{5} m(a^2 + b^2) = \xi \frac{\hbar^2}{2E_I} I(I+1) \qquad (7)$$

where the mass of nucleus is related to its constituents nucleons, $m = Zm_p + (A-Z)m_n$ and ξ is introduced as a correction parameter to compensate for the lack in between the quantum analog and classical expression of the value of moment of inertia ϑ, which is obviously compensates for mass fraction [16] and energy deviation from the rotational spectrum [17]. The nuclear volume *V*, on the other hand, is presumably preserved [16] i.e.,

$$\frac{3}{4\pi} V = R^3{}_0 = a^2 b \qquad (8)$$

where Eqs (6), (7), and (8) can be solved for threeunknowns namely: *a*, *b*, and ξ.

2. Results

The determination of the three parameters *a*, *b*, and ξ can be obtained straight forward by employing the first exited energy state of the ground state band $E(2^+)$, intrinsic electric quadrupole moment Q_0, and the nuclear radius R_0. In Table 1, we present the results of our calculations along with the parameters determined. A sample of our result is compared with other available calculated radii of $^{180,182,184,186}W$ isotopes and are shown in Table 2. The deformation parameter β [10], [15], [18], [19] is calculated and shown along with different sets of earlier calculations in Figure 1.

Table 1. The calculated radii a_{Calc} and b_{Calc}, along with the correction paramet

No	*A*	*Z*	Nucl	*E* **[keV] [18]**	Q_0 **[b] [18]**	a_{Calc} **[fm]**	b_{Calc} **[fm]**	ξ
1	150	62	Sm	333.869	3.684	6.801	5.604	6.253
2	152	62	*Sm*	121.782	5.9	7.118	5.184	2.308
3	154	62	*Sm*	81.976	6.62	7.242	5.074	1.587
4	166	72	*Hf*	158.5	5.93	7.18	5.564	3.49
5	168	72	*Hf*	124	6.57	7.274	5.486	2.78
6	170	72	*Hf*	100.8	7.3	7.379	5.395	2.302
7	172	72	*Hf*	95.22	6.7	7.334	5.525	2.22
8	174	72	*Hf*	90.985	7	7.39	5.505	2.162
9	176	72	*Hf*	88.351	7.28	7.444	5.489	2.138
10	178	72	*Hf*	93.18	6.961	7.43	5.571	2.3
11	180	72	*Hf*	93.326	6.85	7.44	5.619	2.348
12	180	74	*W*	103.557	6.53	7.387	5.701	2.609
13	182	74	*W*	100.106	6.5	7.406	5.734	2.57
14	184	74	*W*	111.208	6.16	7.393	5.818	2.911
15	186	74	*W*	122.33	5.93	7.392	5.883	3.265

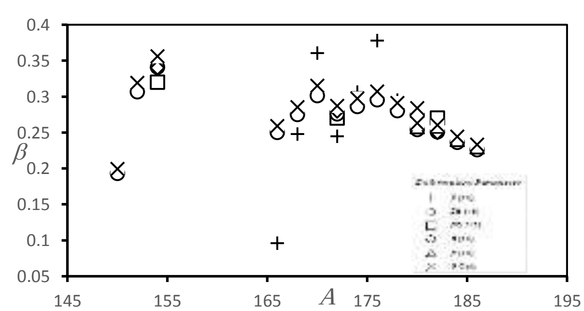

Fig. 1. Comparison of deformation parameter β for different sets of calculations.

Table 2. Calculated radii a_{Calc} and b_{Calc} for *W* isotopes compared with earlier results.

A	*Z*	Nucl	*a* [fm] [20]	*b* [fm] [20]	a_{Calc} [fm]	b_{Calc} [fm]
180	74	*W*	7.33	5.79	7.387	5.701
182	74	*W*	7.36	5.81	7.406	5.734
184	74	*W*	7.32	5.87	7.393	5.818
186	74	*W*	7.3	5.92	7.392	5.883

3. Conclusion

In this work we introduced some correction parameter ξ which we assumed it covers the gap between the classical expression and the quantum related equation expressive for the moment of inertia of even-even nuclei. The calculation we presented are quite encouraging compared with data and other earlier approaches. We noted that the sample isotopes *W* are all deformed in about 1 fm variation from the average radius of the semi-major and semi-minor radius. The calculated values of deformation parameter β is compared with different calculations and presented schematically for convenience.

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