UTILIZING EXCEL SOLVER FOR COMPLEX PIPE NETWORK ANALYSIS AND DEVELOPMENT OF AN EDUCATIONAL TOOL

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الملخص

تهدف هذه الدراسة إلى تحسين منهجية التحليل لشبكات الأنابيب المعقدة. حيث تم اقتراح مقاربة منهجية تتضمن صياغة معادلات الاستمرارية ومعادلات فَقْد الضغط من خلال تطبيق كل من أداة (Excel Solver) وطريقة (Hardy Cross). علاوة على ذلك ساعد استخدام الوظيفة الإضافية (Solver) بمساعد لغة البرمجة (Visual Basic for Application) في توسيع نطاق الدراسة لتتجاوز التحليل النظري، حيث تم تطوير أداة تعليمية وبالتالي توفير موارد عملية لتنمية القدرة على فهم طرق تحليل تدفق عبر شبكات الأنابيب. أيضاً بما أنه تم إجراء تحسين لتحديد قيم معدلات التدفق فقد لوحظ انخفاض كبير في الوقت اللازم لتطبيق النهج التكراري دون أن يؤثر ذلك على دقة الحساب. من ناحية أخرى، قامت الأداة التعليمية المطورة باستكشاف وشرح تحليل أنظمة الأنابيب البسيطة والمعقدة بسهولة وفعالية.

ABSTRACT

This study aims to enhance the analysis methodology for complex pipe networks. An approach was proposed involving the formulation of continuity and head loss equations adapted through the application of both Excel Solver and the Hardy Cross methods. Furthermore, using Solver add-in with the aid of Visual Basic for Application (VBA) helped to extend the study beyond theoretical analysis where an educational tool has been developed, thereby providing a practical resource for further exploration and understanding of such analyzing methods. As the optimization has been conducted and the flow rates values determined, it was observed that there is a significant reduction in the time required for iterative approach while the calculation accuracy was not affected. On the other hand, the developed educational tool has easily and effectively explored the analyzing of simple and complicated piping systems.

KEYWORDS: Excel Solver; Pipe Network Analysis; Hardy Cross Method; Visual Basic for Application (VBA).

INTRODUCTION

One basic and vital talent in engineering is the capacity to solve the problems in an organized and effective manner. Numerous engineering problems have been resolved with the help of tools. Software and programs in particular have been crucial in helping to solve a wide range of problems. Spreadsheet software like Microsoft Excel is not only a valuable but also a favored option since it offers simplicity and availability, even if many software tools have been developed specifically for this purpose. Excel is notable when in contrast to specialized computer software that offers flexibility, integrated features, and an intuitive user interface. Numerous activities, such as data analysis, optimization, modelling, and simulation for diverse applications, can be accomplished with it. Including resolving pipe network issues and other systems of linear and nonlinear equations.

For the safe and reliable transportation and distribution of resources like water and gas, pipe networks are crucial. Liquids and gases flow freely and uninterruptedly inside the tubes, ensuring a steady and uninterrupted supply of fluids. Thus, the role of pipe networks extends beyond mere transportation - they are a critical component in ensuring the sustainability and resilience of urban infrastructure [1]. To ensure the efficient supply of water or gas, the fluid flow through the interconnected pipes is essentially analyzed where the flow rates and the pressure drops in each section should be determined, for more see ref. [2]. The methods developed for the analysis and design of such systems, are discussed in the following section.

The determination of the steady-state flow rate distribution within pipe networks accomplished through the application of one of the specifically devised methods. These include the Hardy Cross method [3], the Newton-Raphson method [4], and the linear theory method [5]. These methods solve a system of linear and nonlinear algebraic equations, by employing an iterative process, striving towards convergence of solution, provided it is attainable. The inputs and outputs in the network are known, but the flow inside the network is unknown. These methods may necessitate considerable time and effort to derive optimal solutions for both the linear and nonlinear sets of equations through the repetition of the required iterative procedure. Furthermore, in the pedagogical context, students learn better by developing the ability to use their knowledge and skills in different situations. [6]. The emergence of computer technology has facilitated the development of software packages and algorithms designed to perform complex calculations. These packages and algorithms enhance pipe network flow analysis by calculating or optimizing solutions for a given set of equations, automating the iterative procedure. Currently, these software programs are commercially available [7].

The existing literature demonstrates the application of the ‘Generalized Reduced Gradient (GRG) Method’, incorporated within an add-in feature of Excel spreadsheets, labelled as ‘Solver’. This Solver is employed for the resolution of nonlinear problems by optimizing data within spreadsheets. The process entails the discovery of an optimal value, performed for one or more target variables, under certain constraints, one or more variables are iteratively modified, adhering to the specified constraints, until the most favorable values for the target variables are identified. Notably, the implementation of this technique is accessible, through Excel, and does not necessitate an understanding of mathematical concepts.

The techniques, delineated in the literature, employ the previously mentioned methods to formulate a system of equations, derived from the principles of mass conservation and fluid mechanical energy. Then, the Solver’s potential to solve nonlinear equations is utilized, akin to the approach presented by Couvillion et al. [8], where an elementary pipe network example was examined. This network, devoid of any pump or turbine, consists of 2 loops, 5 lines, and 4 nodes. The approach, which adhered to the Hardy Cross method, resulted in a solution. Similarly, Adedeji et al. [9] adopted a comparable approach using the Hardy Cross method on a small-scale network, devoid of any pump or turbine, consists of 1 loop, 4 lines, and 4 nodes. As with Sil et al. [10] for a network, devoid of any pump or turbine, consists of 2 loops, 6 nodes, and 7 lines following the Hardy Cross method. Khazaei [11] adopts the Linear Theory Method (LTM) for a network, devoid of any pump or turbine, consists of 4 loops, 12 lines, and 9 nodes. The approach utilizes LTM, introduced additional steps into the procedure necessary for the linearization of the head loss equations. Beyond the analysis of closed loops, pseudo loops were also examined as per Ismail et al. [12]. The authors present the analysis of two pipe networks, employing the Linear Theory Method (LTM) for transforming the non-linear head loss equations and pump characteristic curve. Each network is equipped with a pump, and consists of 1 pseudo loop, 4 lines, and 4 nodes. The study disclosed that 'there is no significant difference between the flows expected and flows calculated'. And in a broader context, various engineering problems, similar to the one described in Rivas et al. [13] including a pipe network consisting of 13 nodes and 16 pipes, absent of any pump or turbine, resolved by determining vectors of the unknown variables.

The other viable step is to utilize such techniques to improve undergraduate fluid mechanics education, as referenced in Huddleston et al. [14], the authors suggest the use of Excel, enabling students to focus on the engineering system and design issues. Brkic [15] suggested utilizing Excel Solver for students' design projects, for the optimization of pipe diameters and the calculation of flow rates following the linear theory and a modified Hardy Cross method [16]. The aforementioned literature works provide guidance in the field of fluid mechanics education, while some suggest a more engaging approach through programming. For instance, El-Awad et al. [17] illustrates how Microsoft Excel, its Solver add-in, and the associated Visual Basic for Applications (VBA) language can serve as an educational platform for the design analyses of fluid-thermal systems.

The above literature review identifies a potential gap in the relevant research. A significant portion of the existing literature emphasizes the ability of spreadsheets in optimizing solutions by solving simple pipe networks. Therefore, this research intends to priorities the optimization of a more complex pipe network and focuses on fulfilling the aim of reducing the time required for manual calculations without compromising accuracy. Additionally, this study aims to create an educational tool utilizing Visual Basic for Applications (VBA). This tool, designed with a user-friendly interface, is intended for undergraduate students who wish to deepen their understanding of pipe network analysis methods.

METHODOLOGY

This section describes the case study network and finding places in the common analysis approach; where changes can be made to increase speed and keep the accuracy. This can include adjusting the iterative procedure, figuring out flow rates. After that, mathematical modelling and programming will be involved to create the improved approach. In order to identify optimal solutions for complicated problems, the improved method should be adapted so that it works well with Excel Solver. The next step is the development of educational tool explaining the improved method and its applications once it has been implemented and validated.

CASE STUDY

As depicted in Figure (1), water distribution system for an industrial manifold was considered. This network comprises of 8 loops, 17 nodes, and 24 lines [18].

The network is subjected to a flow rate of 218 cubic meters per hour (m^3/h), 960 gallons per minute (GPM), pumped into the system. To balance the head loss and adjust the assumed flows, the Hardy Cross method is employed. This iterative process is compiled using Solver, to aid in determining the optimal flow rates within the network.

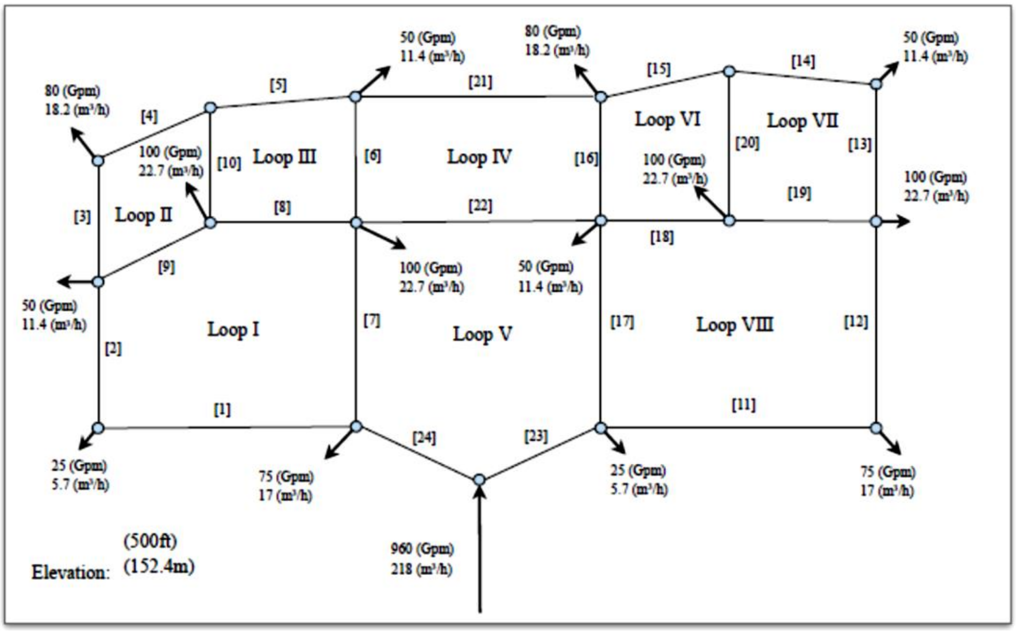


Figure 1: illustration of the considered water distribution system.

ANALYTICAL MODEL

Hardy Cross proposed two methods of analysis. In one of these the total change of head around each loop always equals to zero, and the flows in the pipes of the loop are successively adjusted so that the total flow into and out of each junction finally approaches or becomes zero [3].

$$\Sigma h = 0 \tag{1}$$

Where h is the change in head in conjunction with the flow within any length of pipe:

$$h = rQ^n \tag{2}$$

In this context: ‘r’ represents the loss of head in the pipe per unit flow Q. The value of ‘r’ depends on the length and diameter of the pipe, as well as its surface roughness; and ‘n’ is the flow exponent, and its value varies based on the relationship used to calculate the head loss [19].

The method is underpinned by the principle that the resistance to a change in flow within any pipe is approximately ‘$nrQ^{(n-1)}$’ and establish a counterbalancing flow in each loop to balance the head in that loop to make:

$$\Sigma rQ^n = 0, \text{ equal to } \Delta = \frac{\Sigma rQ^n \text{ (with due attention to direction of flow)}}{\Sigma nrQ^{(n-1)} \text{ (without reference to direction of flow)}} \tag{3}$$

where Δ represents the correction of flow for each loop (Δ1 for loop 1, Δ2 for loop 2, and so on). The procedure is repeated until the desired precision is achieved.

The primary objective of the optimization process is to minimize the total head loss around each loop, as given by equation (2) ‘$h = rQ^n$’ an initial flow, denoted as ‘Q_0’ is assumed for each pipe. This flow should satisfy the continuity equation at each node for each loop. Where the value of ‘n’ is constant [20] and equal to 2, because it is derived from the Darcy-Weisbach equation [21,22].

The correction of flow for each loop is added to the assumed initial flow assumption $Q_0 + \Delta$, for all the pipes in the loops [$(Q_0)_1 + \Delta 1$ for pipe 1 in loop 1, $(Q_0)_2 + \Delta 1$ for pipe 2 in loop 1, and so on], if the pipe is a shared element between two loops, the value is subtracted of the adjacent loop correction. For an example, if pipe 1 were between loop1 and loop2, then the calculation would be $(Q_0)_1 + \Delta 1 - \Delta 2$. The detailed description of the method can be found in references. [23,24].

THE EXCEL MODEL

The initial page of the Excel workbook is presented in Figure (2). It is created to house the fundamental system data, including the initial flow rate assumptions Q_0, resistance r, and the iterative solution's flow rate Q. It also contains the head losses for all lines across all loops. The analysis commences with a flow rate assumption that satisfies continuity where the corrections for each loop's flows Δ are also displayed on this page.

Loop	Pipe	Q0	r	Q	h	Σh	Loop	Pipe	Q0	r	Q	h	Σh
I	1	0.44560	311.90130	0.44560	61.93148	-101.54642	V	7	0.45674	148.20510	0.45674	30.91755	16.50848
	2	0.38990	148.91110	0.38990	22.63800			22	-0.02228	7273.50400	-0.02228	-3.61056	
	7	-0.45674	148.20510	-0.45674	-30.91728			17	-0.28964	150.51010	-0.28964	-12.62649	
	8	-0.18938	5438.27500	-0.18938	-195.04256			23	-1.06944	16.12213	-1.06944	-18.43891	
	9	0.07798	6552.33100	0.07798	39.84394			24	1.06944	17.72030	1.06944	20.26689	

Loop	Pipe	Q0	r	Q	h	Σh	Loop	Pipe	Q0	r	Q	h	Σh
II	9	-0.07798	6552.33100	-0.07798	-39.84394	-44.22409	VI	16	0.08912	3990.65500	0.08912	31.69528	34.20835
	3	0.20052	124.00450	0.20052	4.98606			15	0.06684	4522.22400	0.06684	20.20342	
	4	0.02228	4917.35900	0.02228	2.44097			20	0.04456	5946.41900	0.04456	11.80717	
	10	-0.04456	5946.41900	-0.04456	-11.80717			18	-0.06684	6602.56500	-0.06684	-29.49752	

Loop	Pipe	Q0	r	Q	h	Σh	Loop	Pipe	Q0	r	Q	h	Σh
III	10	0.04456	5946.41900	0.04456	11.80717	214.71603	VII	20	-0.04456	5946.41900	-0.04456	-11.80717	133.15999
	5	0.06684	6922.61800	0.06684	30.92739			14	0.11140	6770.16400	0.11140	84.01746	
	6	-0.06684	5162.32900	-0.06684	-23.06315			13	-0.22280	152.27270	-0.22280	-7.55879	
	8	0.18938	5438.27500	0.18938	195.04462			19	0.11140	5520.44400	0.11140	68.50849	

Loop	Pipe	Q0	r	Q	h	Σh	Loop	Pipe	Q0	r	Q	h	Σh
IV	6	0.06684	5162.32900	0.06684	23.06315	-0.62547	VIII	11	-0.72410	307.79020	-0.72410	-161.38081	-233.50846
	21	0.02228	8855.98900	0.02228	4.39610			12	-0.55700	147.44050	-0.55700	-45.74327	
	22	0.02228	7273.50400	0.02228	3.61056			19	-0.11140	5520.44400	-0.11140	-68.50849	
	16	-0.08912	3990.65500	-0.08912	-31.69528			18	0.06684	6602.56500	0.06684	29.49752	
								17	0.28964	150.51010	0.28964	12.62658	

Δ1	
Δ2	
Δ3	
Δ4	
Δ5	
Δ6	
Δ7	
Δ8	

Figure 2: Excel workbook page for optimization.

Solver is accessible from Excel's Data tab where it can optimize a target cell's value by altering the values of the cells used in its calculation. Figure (3) illustrates how Solver is configured to determine flow rates by adjusting the flow correction values. The goal is set to the sum of the head losses in loop 1 to be equal to zero. Solver enables the user to apply constraints to the solution, and Figure (3) also displays eight constraints, which are the sums of the head losses in all loops. It's worth mentioning that Solver providing the GRG Nonlinear method is used for solving nonlinear problems. It looks at the gradient or slope of the objective function and is highly dependent on the initial conditions.

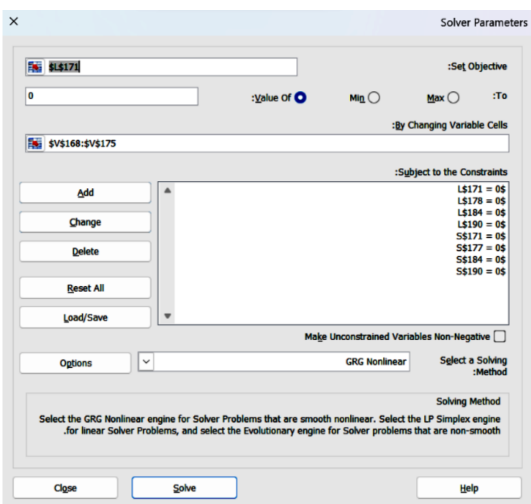


Figure 3: Excel Solver parameters configuration.

THE EDUCATIONAL TOOL

In the creation of the educational tool, Visual Basic for Applications (VBA) was employed to extend the functionality of Excel through programming. This was particularly useful in crafting educational materials that offer an in-depth understanding of pipe flow analysis methods, with a focus on the Hardy Cross method. These materials, encompassing tutorials and example problems, aim to clarify both the theoretical and practical aspects of the method. Through tutorials, example problems, and step-by-step guides, the educational materials aim to provide a comprehensive understanding of both the theory and practical applications of the pipe flow analysis methods and Excel.

The next set of figures illustrates the navigation through different windows of the educational tool, displaying fluid mechanics principles, Figure (4-a), problem visualization, Figure (4-b), for the systematic guides, were the figure shows an example familiar to students from a textbook [25] related to undergraduate network flow topic.

Another feature of the tool is a section where the information about the problem is located. The data of the problem can be entered or changed, to aid in understanding the effects of different variables. Upon completion, the chart refreshes instantly when new data is added, and students can export the obtained results, Figure (4-c), and the corresponding chart, Figure (4-d), to a Portable Document Format (PDF) file.

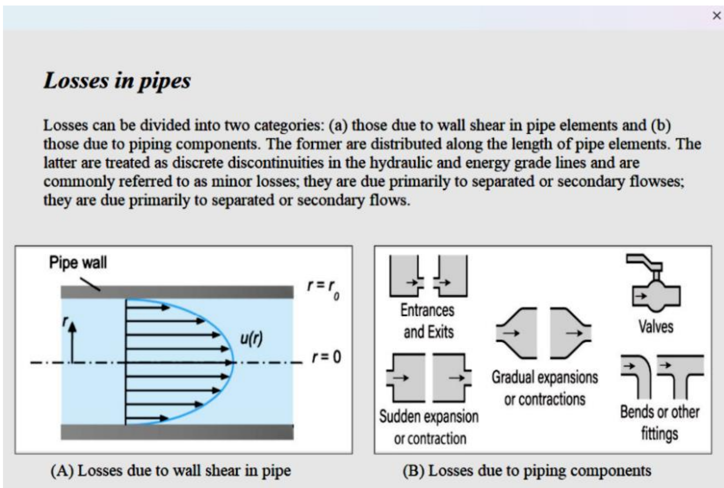


Figure 4-a: Fluid mechanics principles window.

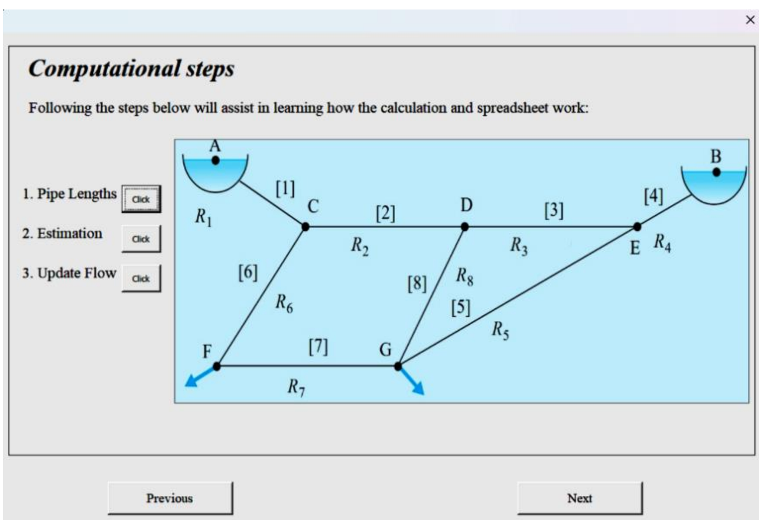
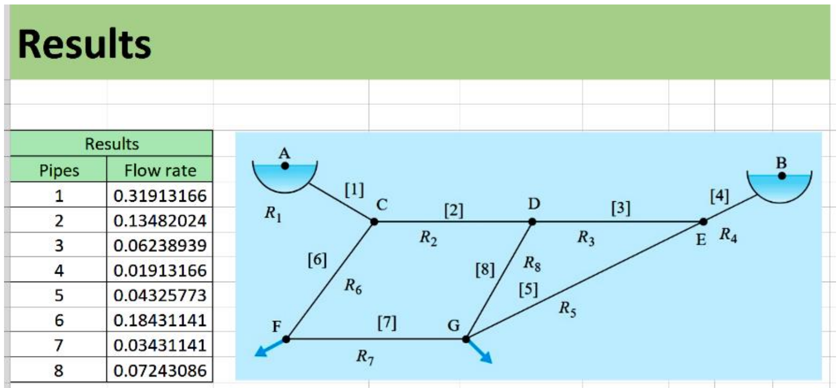


Figure 4-b: The displaying window for simple pipe problem.



Results	
Pipes	Flow rate
1	0.31913166
2	0.13482024
3	0.06238939
4	0.01913166
5	0.04325773
6	0.18431141
7	0.03431141
8	0.07243086

Figure 4-c: Results obtained.

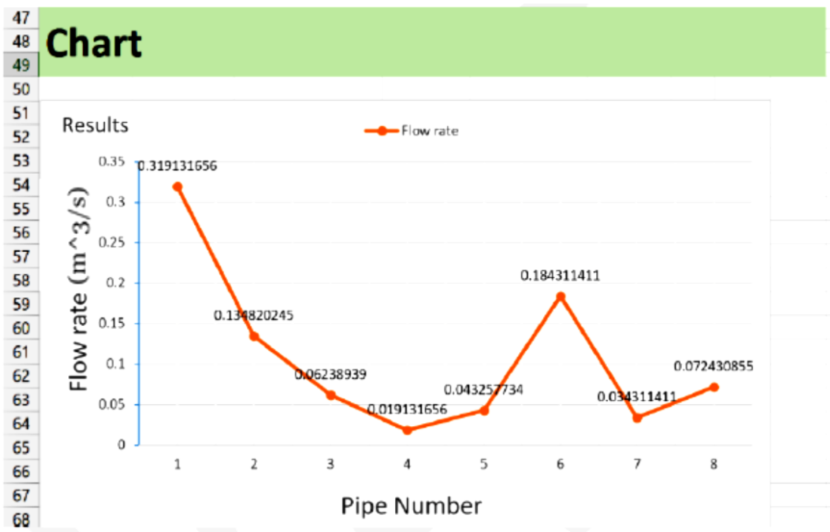


Figure 4-d: Graphical representation.

RESULTS AND DISCUSSION

The results of this study will be presented in two sections. First, the findings related to the analytical application of Excel in addressing the presented case study are to be explored. Second, the implications on the educational field by presenting the results acquired from the application of the educational model on the complex network that we considered in the current work.

The results obtained, presented in Table (1), compare the flows using the Hardy Cross method with Solver technique and the conventional Hardy Cross method. It can be noticed that both methods start with a similar initial flow estimate that satisfies the principle of continuity at the nodes. Accordingly, drawing a comparison between the two approaches is possible. Moreover, the optimal network solution selected based on the maximum number of iterations needed to achieve an acceptable level of convergence.

Excel Solver was configured with specific settings to optimize the solution process. The Solver was set to run with unlimited iterations and time, allowing it to exhaust all possible solutions. The precision was set at the level of 0.000001, ensuring the accuracy of the solutions obtained. The convergence rate was set at 0.0001which allowing the Solver to efficiently navigate the solution space and provide a balance between solution accuracy and computational efficiency.

During the solution process, the Solver did not encounter any feasibility problems, indicating that the problem was formulated within the capabilities of the Solver. The solution time was remarkably fast, with the Solver arriving at an optimal solution in just 0.5 seconds.

Table 1: Essential Case Study Pipe Network Data and Flow Obtained

Pipe number	Resistance (r)	Initial flow (m^3/s)	Flow obtained using Microsoft Excel Solver (m^3/s)	Flow obtained using Hardy Cross method (m^3/s)
1	311.9013	0.44560	0.38453	0.38550
2	148.9111	0.38990	0.32883	0.32980
3	124.0045	0.20052	0.16352	0.16451
4	4917.359	0.02228	0.01472	0.01373
5	6922.618	0.06684	0.06787	0.06700
6	5162.329	0.06684	0.06841	0.06844
7	148.2051	0.45674	0.23841	0.23860
8	5438.275	0.18938	0.11574	0.11565
9	6552.331	0.07798	0.05392	0.05388
10	5946.419	0.04456	0.05315	0.05327
11	307.7902	0.72410	0.69342	0.69244
12	147.4405	0.55700	0.52632	0.52534
13	152.2727	0.22280	0.27627	0.27563
14	6770.164	0.11140	0.05793	0.05857
15	4522.224	0.06684	0.01936	0.02045
16	3990.655	0.08912	0.17479	0.17497
17	150.5101	0.28964	0.59972	0.59955
18	6602.565	0.06684	0.14499	0.14489
19	5520.444	0.11140	0.02725	0.02691
20	5946.419	0.04456	0.05055	0.05100
21	8855.989	0.02228	0.11087	0.10996
22	7273.504	0.02228	0.16854	0.16829
23	16.12213	1.06944	1.34884	1.34769
24	17.7203	1.06944	0.79004	0.79120

In addition, the graphical representation, which compares the obtained flows illustrated in Figure (5) as follows:

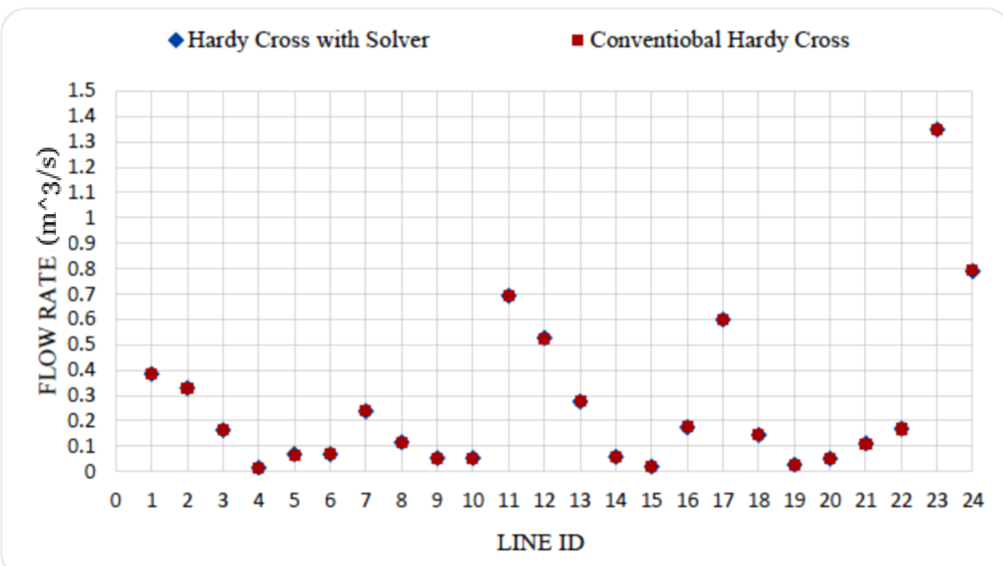


Figure 5: Hardy Cross with Solver and the conventional Hardy Cross Comparison.

Upon initial observation of the previous chart, a noticeable overlap is observed between the Solver values and the conventional values resulted in this study. The regions on the chart exhibiting high flow rates are predominantly situated in proximity to the supply tank. For instance, the maximum flow is observed in pipe 23, which can be attributed to its relatively low resistance (r = 16.12213) in comparison to the adjacent pipes, as well as its proximity to the supply flow within the network. On the other hand, lines 4, 15, and 19 have the lowest flow rates, which can be attributed to their relatively high resistances of 4917.359, 4522.224, and 5520.444 respectively.

Considering the running time of 0.5 second, it took 24 iterations for Solver to find a solution, with all Constraints and optimality conditions satisfied. For the same amount of iterations, the iterative process of the conventional Hardy Cross method took 4 minutes and 42 seconds for convergence of the solution.

Figure (6) shows the iterative progression towards a solution using the traditional Hardy Cross method. Where, the conventional Hardy Cross method progressed towards a solution, initially, quicker and then slowed down as the iterative process converged into a solution. Meanwhile, the optimal solution in using Solver, Figure (7), accelerates its rate of convergence to reach an optimal solution. It is worth noting that the Excel Solver will continue to iterate despite reaching an optimal or nearly optimal solution, in a short amount of time. It also allows for user customization through the setting of tolerance and convergence criteria, which could be considered a potential advantage in scenarios where reliable solutions are needed. A strict convergence criterion may lead to a better approximation of the root, but with an increase in the computational time required.

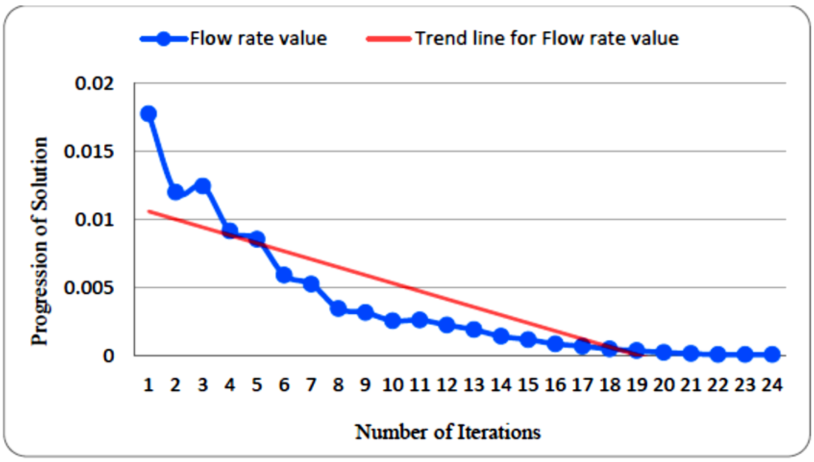


Figure 6: Progression of iterations towards a solution conversion using the conventional Hardy Cross method.

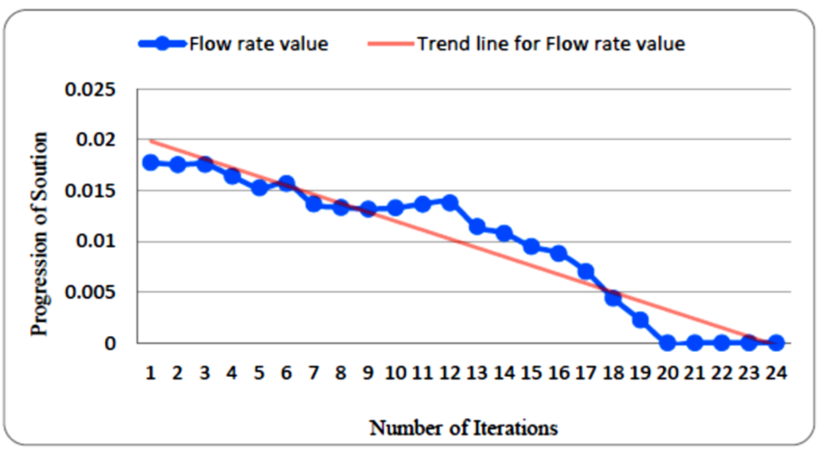


Figure 7: Progression of iterations towards a solution conversion using the Hardy Cross method with Solver.

The developed educational model was designated to handle different pipe networks ranging from simple networks with a few pipes to complex systems with multiple loops and nodes. Each problem will come with detailed solutions, illustrating the step-by-step application of the Hardy Cross method.

The use of Excel VBA code to create an educational tool has shown positive outcomes. This tool enhances learning by offering a range of example problems, from simple to complex network systems, Figure (8-a). Each problem includes detailed solutions, allows data to be inputted into a table and calculations to be performed, as shown in Figure (8-b). The results are then printed in a PDF (Portable Document Format) for preservation and sharing, as shown in Figure (8-c), which displays the written code for the printing function.

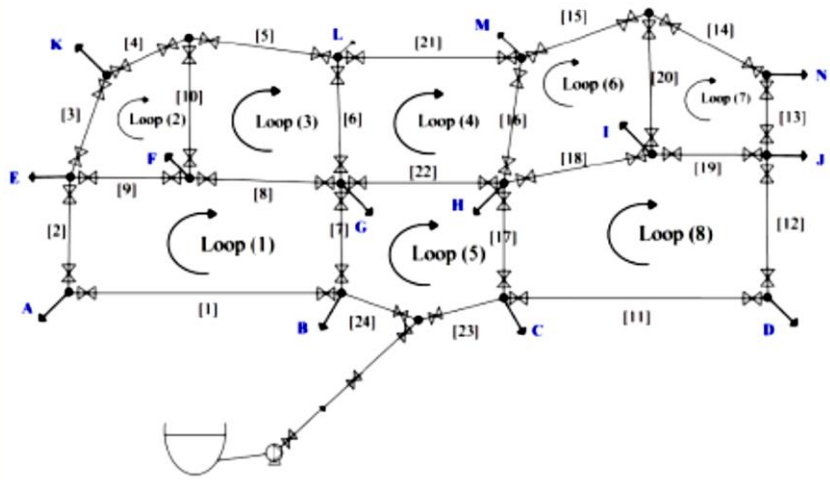


Figure 8-a: Case study network as a complex example for the educational tool.

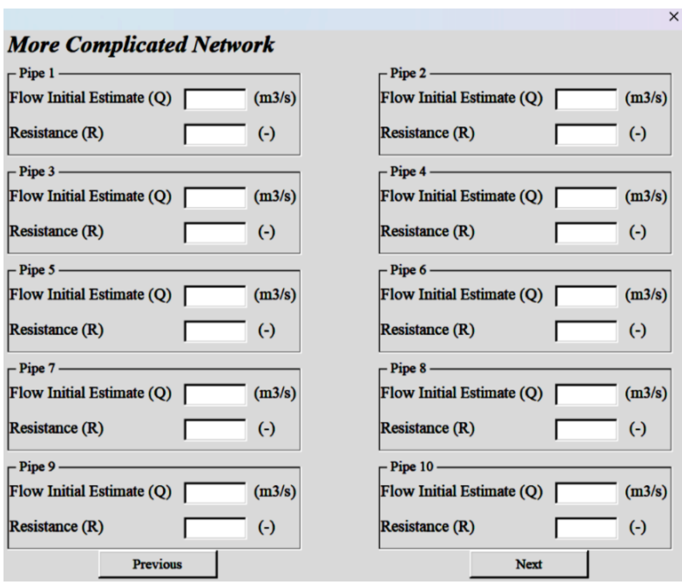


Figure 8-b: Educational tool data-entry interface.

The Solver function is used for analysis, giving students practical insights. Furthermore, the tool effectively improves students' understanding of pipe network analysis methods and has great potential to enhance learning outcomes.

The application of the educational model using Solver on complex network analysis yielded significant results. The model was able to effectively handle the complexity of the network and provide insightful outputs. It demonstrated efficiency, accuracy, usability, and scalability, making it a valuable tool for both educational and practical applications in network analysis.

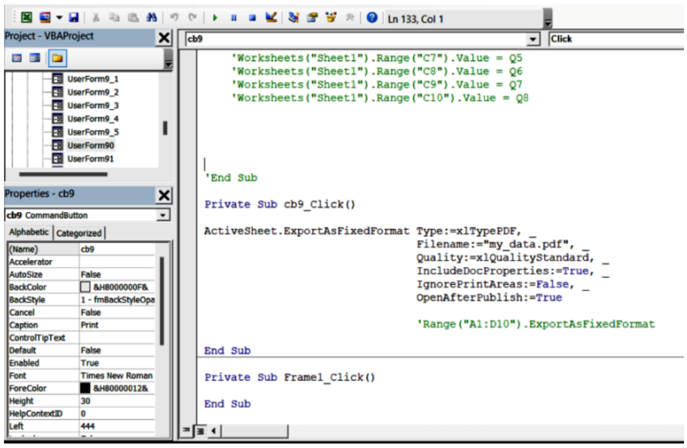


Figure 8-c: Visual Basic code for the printing function.

CONCLUSION AND RECOMMENDATIONS

This research explores the application of Microsoft Excel Solver, a supplementary feature, in tackling pipe network challenges. The scope of the study extends to intricate pipe network analyses. The computation of flows within the pipes was accomplished utilizing both the Microsoft Excel Solver tool and the Hardy Cross method.

The acquired results revealed that Microsoft Excel Solver is a reliable and practical tool, which helps to run the analysis with less time consumption, reducing the human errors and giving extremely reasonable accuracy. Judging by the superior correlation coefficient values, Microsoft Excel Solver proves to be a more efficient alternative than the conventional utilization of Hardy Cross method for this particular application.

Furthermore, The Excel-based educational tool enables students to develop models that integrate theoretical knowledge with computer skills, thereby enriching their learning experience.

List of Symbols and Abbreviations

h - Head; head loss [m]

Q - Flow rate [m^3/s].

Q_0 - Flow rate [m^3/s]

Δ – Flow rate correction [-]

(GRG) - The Generalized Reduced Gradient

REFERENCES

[1] Alternative Energy and Shale Gas Encyclopedia. United Kingdom: Wiley, 2016.

[2] Houghtalen, Robert J., A. Osman Akan, and Hwang Ned H C. *Fundamentals of Hydraulic Engineering Systems*. Boston: Pearson, 2017.

[3] Cross, H., 1936. Analysis of flow in networks of conduits or conductors. *University of Illinois. Engineering Experiment Station. Bulletin; no. 286*.

[4] Spiliotis, M. and Tsakiris, G., 2011. Water distribution system analysis: Newton-Raphson method revisited. *Journal of Hydraulic Engineering*, *137*(8), pp.852-855.

[5] Wood, D.J. and Charles, C.O., 1972. Hydraulic network analysis using linear theory. *Journal of the Hydraulics division*, *98*(7), pp.1157-1170.

[6] Marshall, D., Summer, M. and Woolnough, B., 1999. Students' conceptions of learning in an engineering context. *Higher education*, *38*(3), pp.291-309.

[7] Ormsbee, L.E., 2008. The history of water distribution network analysis: The computer age. In *Water Distribution Systems Analysis Symposium 2006* (pp. 1-6).

[8] Couvillion, R.J. and Hodge, B.K., 2009. A unified approach to piping system problems using Excel. *Comput. Educ. J*, *19*(1), pp.24-33.

[9] Adedeji, K.B., Hamam, Y., Abe, B.T. and Abu-Mahfouz, A.M., 2017, July. A spreadsheet tool for the analysis of flows in small-scale water piping networks. In *2017 IEEE 15th International Conference on Industrial Informatics (INDIN)* (pp. 1213-1218). IEEE.

[10] Sil, B.S., Banerjee, P., Kumar, A., Bui, P.J. and Saikia, P., 2013. Use of excel-solver as an optimization tool in design of pipe network. *International journal of hydraulic engineering*, *2*(4), pp.59-63.

[11] Khazaei, M., 2017. Applying Solver engine in Microsoft Excel to analyze water distribution network based on linear theory method. *Journal of Safety, Environment, and Health Research*.

[12] Ismail, A., Lukman, S., Ojo, S.O., Bolorunduro, K.A., Adeosun, O.O. and Oke, I.A., 2016. Solutions of selected pseudo loop equations in water distribution network using microsoft excel solver. *Ife Journal of Science*, *18*(2), pp.371-390.

[13] Rivas, A., Gómez- Acebo, T. and Ramos, J.C., 2006. The application of spreadsheets to the analysis and optimization of systems and processes in the teaching of hydraulic and thermal engineering. *Computer Applications in Engineering Education*, *14*(4), pp.256-268.

[14] Huddleston, D.H., Alarcon, V.J. and Chen, W., 2004. A spreadsheet replacement for Hardy-Cross Piping System analysis in undergraduate hydraulics. In *Critical Transitions in Water and Environmental Resources Management* (pp. 1-8).

[15] Brkić, D., 2016. Spreadsheet-based pipe networks analysis for teaching and learning purpose. *Spreadsheets in Education*, *9*(2).

[16] Epp, R. and Fowler, A.G., 1970. Efficient code for steady-state flows in networks. *Journal of the hydraulics division*, *96*(1), pp.43-56.

[17] El-Awad, M.M. and Al-Saidi, M.S., 2022. Excel as an Educational Platform for Design Analyses of Fluid-Thermal Systems. *World Journal of Engineering and Technology*, *10*(2), pp.434-443.

[18] محمد صبحي غسان حلي، (2012)، تحليل شبكة مياه مجمع صناعي باستخدام طريقة هاردي كروس وبرنامج EPANET 2.

[19] Fincham, Anthony E. *A Review of Computer Programs for Network Analysis:(Developed at London Research Station)*. Gas Gouncil, 1971.

[20] Stephenson, David. *Pipeline Design for Water Engineers*. Amsterdam: Elsevier Scientific Pub. Co., 1981.

[21] Rennels, Donald C. Pipe flow: A practical and comprehensive guide. John Wiley & Sons, 2022.

[22] Wannapop, Rangsan, Thira Jearsiripongkul, and Krit Jiamjiroch. "Adaptive urban drinking water supply modeling using the effect of node elevation and head loss formula: A case study." *Heliyon* (2024).

[23] Bober, William. "The use of the Swamee-Jain formula in pipe network problems." *Journal of pipelines* 4 (1984): 315-317.

[24] Bober, William, and Richard A. Kenyon. *Fluid mechanics*. New York: Wiley, 1980.

[25] Potter, Merle C., David C. Wiggert, and Bassem H. Ramadan. 2011. *Mechanics of Fluids*.