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THE OPTIMAL ECONOMIC DESIGN OF $\overline{X}-$ CHART WITH RUNS

A thesis submitted to the Department of Statistics in partial fulfillment of the requirements for the degree of Master of Science in Statistics

BY

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Chapter I

Introduction

Human needs must be satisfied through services and products. In the past, resources and products were limited and everything produced was consumed regardless of its condition. Technical developments have led to multiple competitive products, which led to a search for ways to differentiate between them. Many criterions have been used to choose between equivalent products such as price ,shape ,colour , quality ,etc.

Quality of a product is an important consumer decision factor, consequently,it becomes a key of firms success.

The concentration on quality has been increased in the last decay because the improvement of quality ensures the market share, reduces production costs, and hence increases profit.

Quality has many meanings differ from one user to another, from time to time, and from place to place. This makes the definition of quality is the first step for any production process.

1.2-Definitions of quality:

The meaning of quality has been changed during the period of the quality evolution. The following definitions of quality have occurred:

1- Quality has been defined by the European Organization for Quality Control (EOQC) as ″a set of characteristics and advantages that make the product able to meet the specific needs. EOQC confirmed that the quality of manufactured products depend mainly on the quality of design and quality of manufacturing ,where the quality of design represents by facilitating manufacturing processes to meet the requirements of the consumer while the quality of manufacturing means accurately matching the product with the objectives of its design.

2- The American Society for Quality Control has defined quality as: ″the total of all manifestations and properties of the product or service related to its ability to meet the specific need″.

3- International Standardization Organization(ISO) has defined quality as: ″the overall advantages which defining its ability to meet the described or contained needs″**.**

Accordingly, quality has been defined as appropriateness of a product to meet the needs of the consumer with a distinction between the quality of design and quality of manufacturing .

1.3-Quality and variation:

Generally; some differences between the units of the same product may exist due to changes in the process conditions, which might lead to products of different quality. Thus, quality can be deteriorated by the process variation. However, the degree of deterioration depends on the causes of such variation. In general, there are two general causes of the process variation:

1.3.1-Random (natural, normal) causes:

These causes are considered as a part of the process and they cannot be avoided or controlled, their resulting differences are usually of small amounts and do not make big problems for quality.

A process that works with the existence of these causes only is considered to be under control and it does not need intervention to improve it. Examples of such causes are:

A slight change in the measurements, some changes of raw materials, or little vibration in a machine parts, or slight changes of the temperature of the production halls etc.

The variation due to these causes is called random (normal, common) variation.

1.3.2- Assignable (non-random, real , abnormal)causes:

These causes can be easily identified, they often lead to significant differences in the properties of the product and thus substantial problems in quality. Consequently, the process that operates under these causes is considered to be out of control and it must to be corrected. Examples of these causes are:

Defects in the machine, or big changes of the properties of the raw material, great changes in the production halls temperature, etc.

The variation caused by assignable causes is called abnormal(real) variation.

Statistical methods can distinguish between these types of causes, and hence they provide procedures to determine the states of a process. The statistical methods provide many tools for quality control, they are called statistical quality control (SQC) tools. The statistical quality control tools can be divided into three categories:

1-Descriptive statistics:

They are used to describe the quality characteristics and relationships among them such as: mean, standard deviation, range, skewness, kurtosis, coefficient of variation. correlation, regression, etc .

2-Acceptance sampling(AS):

Acceptance sampling refers to the procedure of inspecting a sample that is randomly selected from a large lot of items, and deciding whether to accept or reject the entire lot upon the inspection result.

3-Statistical Process Control (SPC):

It is a procedure to decide whether the process' outputs agree with the requirements or not. Thus, SPC tools determine whether the process is functioning properly or it needs some corrective actions. SPC consists of several tools, control charts are the most applicable among them.

1.4-Control Charts:

Control charts are the most important widely used statistical tool for process control. They control the variability of the quality characteristic of interest via introducing bands for the normal (common) variability, and any variability beyond that is considered abnormal, and its process may require some correction actions. The construction of a control chart requires the following steps:

1. Determination of the quality characteristic of interest.
2. Determination of the appropriate statistic for the characteristic of interest.
3. Determination of the control limits, the sample size and the sampling interval .

4. Calculating the value of the statistic of interest.

5. Drawing the control chart.

This type of chart is called Shewhart chart because it is constructed according to the following Shewhart principals.

1. 4.1-Shewhart principals of construction a control chart:

Shewhart in 1924 introduced for the first time the concept of statistical control charts when he authored a book entitled " economic control of the quality of manufactured products ".

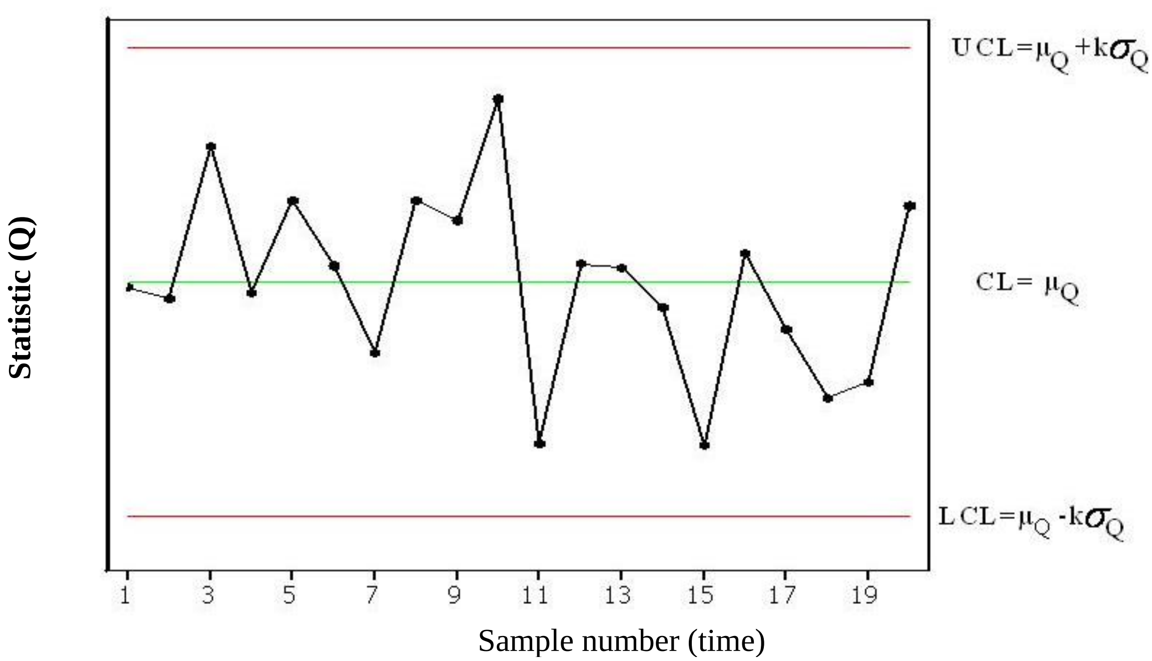
He described a procedure for quality control of production process through the design of chart called the chart of Shewhart to control quality, which is composed of: upper control limit (UCL) and lower control limit (LCL) represent the ceiling and the floor respectively of the normal variation of a statistic that represents the quality characteristic under study , and a central line (CL) indicating its desired level . Upon plotting a series of values of the statistic, any point out of these limits is taken as evidence of the systemic causes of the change in the production process .

Shewhart principals for constructing control charts can be described as follows:

Let Q be the statistic of interest which has normal distribution with mean μ_Q and variance σ_Q^2 .Then the center line is CL=μ_Q which represents the target level of the statistic of interest, LCL=$\mu_Q - k\sigma_Q$ and UCL= $\mu_Q + k\sigma_Q$ where k is the distance between the control limits and the center line expressed in term of the standard deviation of Q .

Upon a periodical drawing of samples of given sizes and plotting the sample statistic (Q) on the chart, a decision can be made about the state of the process.

Consequently, statistical control charts are diagrammatic graphics (as shown in the figure below) to trace the stages of the process and indicates the changes in the quality of the production process over periods of time. So, they provide a way to distinguish between the natural variation resulting from random causes and the variation due to assignable causes which must be removed in order to correct the process. Therefore, the most important use of control charts is to control the quality of products because they give indications whether there is a deviation from the standards (specifications) for the product.



Shewhart Chart (1.1)

1.4.2-Types of Control Chart:

Control charts are of two categories according to the nature of the statistic that represents the quality of interest.

1) Variables control charts:

These charts are used to control certain features(characteristics or properties) of the quality characteristic of interest that can be measured on a continuous scale such as height , weight , speed , etc.

Among these charts we mention: $\overline{X}$ –chart, S-chart, R-chart .

2)Attribute control charts:

These charts can be used to monitor characteristics that cannot be measured numerically but they are subject to certain judgments such as good or bad, valid or invalid. Examples of these charts are: p-chart, np-chart, c – chart.

Any control chart can be obtained from the general Shewhart principals by replacing Q with the statistic of interest, for example; replacing the statistic Q by $\overline{X}$ gives the $\overline{X}$-chart and replacing Q by p given p- chart.

1.5- $\overline{X}$ - Chart:

$\overline{X}$ - chart is the most commonly used tool to control production process. In accordance with the principles of the Shewhart, let $(x_1, x_2, x_3, ..., x_n)$ be a random sample from a process whose characteristic of interest has $N(\mu_0, \sigma^2)$, then $\bar{X} \sim N\left(\mu_0, \frac{\sigma^2}{n}\right)$, where μ_0 is the desired process level and σ^2 is the process known variance, then the chart has:

LCL=$\mu_0 - k\frac{\sigma}{\sqrt{n}}$, CL=$\mu_0$, and UCL=.$\mu_0 + k\frac{\sigma}{\sqrt{n}}$

Thus, the chart parameters are (n, k, h),where n is the sample size , k is the width of control limits, and h is the sampling interval. In order to use this chart, we must determine values of (n, k, h) which is usually called the design of the chart.

1.6-The design of $\bar{X}$ -Chart:

The design of $\bar{X}$-chart means the determination of the values of the parameters (n, k, h) **.** There are many criterions for the design of control charts:

1-The statistical design:

The utilization of this design is planned to find the value of (n, k, h) to satisfy certain statistical properties of the chart, such as to give specific values of the probabilities of type I and type II errors.

It should be mentioned that this procedure does not take the economic consequences into its consideration.

2-The economic design:

The use of a chart has some economic consequences such as the costs of Sampling, the costs of testing, the costs associated with the investigation of outside of control signals, and costs of possible of correction of the process. Taking these economic consequences into consideration when the values of the parameters (n, k, h) are to be determined is called the economical approach to design control charts.

This approach faces some criticisms such as it does not take account of statistical properties of the chart.

3-Economic statistical design:

This approach refers to the economic design of a chart while taking the statistical properties of the chart as constraints.

The $\overline{X}$ -chart is widely used to control production processes, but it is insensitive for small deviations from the target level. To increase the chart sensitivity for the detection of small shifts the following rules are introduced.

1.7-Sensitizing rules:

There are some rules to increase the sensitivity of the chart to detect (small) shifts such as:

- Finding one point outside k- standard deviation control limits.

- Finding of two consecutive points between the control limits and the warning limits
- Finding 2 out of 3 consecutive points outside two standard deviation control limits.
- Finding 4 out of 5 consecutive points outside one standard deviation control limits.
- Run of six or more consecutive points above or below the control line (CL) and within the control limits.
- Run of fourteen or more consecutive points in up/ down saw-tooth pattern.
- Nonrandom pattern on the chart.

1.8-Statement of the problem:

The $\overline{X}$-chart is the most commonly used chart to monitor the level of a specific property of the output of a production process. Economic design of control $\overline{X}$ -chart has a vast area of researches. The chart has been found to be insensitive to detect(small) deviations from the desired level of quality characteristic interest. Increasing the sensitivity of the chart can be done by several procedures including the introduction of a run of 8 consecutive points above or below the central line (CL) and within the control limits. This procedure will lead to a chart called $\overline{X}$ -chart with a run of 8 points. This chart has CL= μ_0, LCL= $\mu_0 - k\frac{\sigma}{\sqrt{n}}$, and UCL= $\mu_0 + k\frac{\sigma}{\sqrt{n}}$, and its parameters are (n,k,h). where, n is the sample size, h is the sampling interval, k the control limits.

To use this chart, its parameter (n,k,h) must be determined from the economic point of view which is the object of this study:

"The optimal economic design of $\bar{X}$ -chart with a run of 8 points" .

.19-Previous studies:

There are several researchers studied the issue of the economic design of the $\bar{X}$-chart. Duncan (1956) has proposed a model for the optimal economic design of the control $\bar{X}$-chart, His paper was the first to deal with the full economic model for Shewhart type control charts. Taylor (1968) has clarified that control chart with fixed sample size at fixed periods of time are not ideal. He derived optimal control rule regarding two processes, with the normal distribution of the properties of quality characteristic of interest.

It seems that the cost structure of Duncan's model is more realistic than that of Knappenberger and Grandage model. Also, Knappenberger and Grandage model allows the continuation of deterioration of the quality after the initial change, which may be the most realistic then that of Duncan's model. Harrington in (1997) made several attempts to study the importance of developing the optimal economic model to improve the production process by discovering the actual causes for the cases of out of control. Woodall (1986) criticized the economic design of control charts. In addition, Del Castillo et al. (1996) went further with a multi-criteria approach that avoids explicitly the costs of false alarms and running out of control. Chiu and Cheung (1977) formulated an economic model of the $\overline{X}$-chart with warning limits using the expected cost per unit time as the objective function. However, none of them gave an explicit formula to determine the required parameters. The economic design of $\overline{X}$-chart with warning limits has been discussed by several authors. In all cases, researches have been confined to single assignable cause systems. For example, Tiago de Oliveira and Littauer (1965,1966) have developed a procedure for selecting the parameters of the control chart on an economic basis. They determine an economically optimal sample size and sampling interval. But not in a monotonic manner(Del Castillo, 2001). In a vast majority of Quality Control applications. There are multiple quality characteristics of interest. Frequently, there are multiple controllable factors that can be manipulated to modify the quality characteristics. This underlines the practical relevance of finding multivariate process adjustment techniques.

CHAPTER II

Formulation of the objective function

2.1-Introduction

The $\overline{X}$-chart is characterized by its simplicity and quick detection of large shifts. The main drawback of this chart is its lack of sensitivity to small shifts in the process' level which makes the chart inefficient. This study is proposed to increase the sensitivity of the $\overline{X}$-chart by using a run of 8 consecutive points above or below the central line and within the control limits.

The design of chart has some economic sequences which should be taken into account. Hence this study is an economical approach to design $\overline{X}$- chart with runs. The approach requires the formulation of objective function (the loss per produced unit) which links the costs and benefits of charting to the chart parameters. The formulation objective function needs a description of the mathematical model of the charting procedure which includes the process behavior, the statistical properties of the $\overline{X}$- chart with runs and the economic consequence of the charting.

2.2: The mathematical model of charting:

The formulation of the objective function which relates the control procedure to its economic consequences requires detailed information and assumptions about the process behavior, the statistical properties of the chart, and the profit and cost parameters. These aspects will be discussed in the following sections.

2.2.1- The process behavior:

The process under consideration is assumed to be characterized by:

1. The production is conducting at rate of ν units per hour.
2. The quality characteristic of interest (X) is a random variable has a normal distribution with mean μ and variance σ^2, that is $X \sim N(\mu, \sigma^2)$, thus $\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.
3. The process consists of a series of cycles, each cycle begins with the process in state I (the process under control) with a mean of μ_0 and ends up by detecting that the process is in state II (the process out of control) with a mean of $\mu \neq \mu_0$ due to the occurrence of an assignable cause which must be corrected in order to return the process to state I, and then , a new cycle begins.
4. The occurrence of the assignable cause shifts the process to state II with mean $\mu_1 = \mu_0 - \delta\sigma$ with probability p ($\mu = \mu_1$) or $\mu_2 = \mu_0 + \delta\sigma$ with probability of

$p\,(\mu = \mu_2)$ where δ (shift size) and σ are known.

5. The duration of the in control state (state I) is a random variable (T) which has an exponential distribution with parameter λ, so the expected value of the time for under control state is $E(T) = \frac{1}{\lambda}$ hours

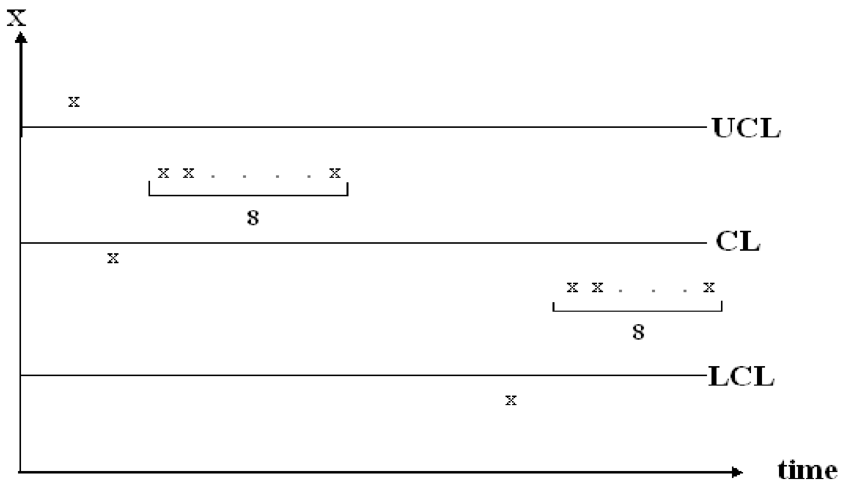
6. The states of the process can be detected by charting only.

7. The process is not self correcting.

2.2.2 The statistical characteristic of the chart:

The $\bar{X}$-chart with runs of 8 consecutive points has central line $CL = \mu_0$, and upper control limit , $UCL = \mu_0 + k\frac{\sigma}{\sqrt{n}}$,and a lower control limit, $LCL = \mu_0 - k\frac{\sigma}{\sqrt{n}}$ where μ_0, σ are known. The chart parameters(k,n,h) are to be determined from economic point of view.

The control procedure composed of a cyclic drawing of samples of size (n) at interval of (h) hours from the process under investigation . A sample point $\bar{X}$ that falls outside the control limits $[LCL, UCL]$, or 8 consecutive sample points $(\bar{x}_{j-7}, \bar{x}_{j-6}, \ldots\ldots, \bar{x}_j)$ fall on one side of control line and with the control limits are taken as an indication of out of control condition which might require correction, as illustrated in figure(1.2) below.



(1.2) $\overline{X}$ -chart with run of 8.

However, the are two possibilities of wrong signals.

1-The false alarm (type-I error):

A false alarm occurs when the process is under control but the chart gave wrong indication of an out of control state. The probabilities of its occurrence are:

$\alpha_1 =$ probability of out - of - control signal from the j^{th} sample (j < 8) when the process is under control

$$\alpha_1 = p(\overline{x}_j \notin [LCL, UCL] \| \mu = \mu_0) = 2\,\Phi(-k) \qquad (2.2.2.1)$$

$\alpha_2 =$ probability of out-of-control signal from the j^{th} sample ($j \geq 8$) when the process is under control

$$= p(\overline{x}_j \notin [LCL, UCL] \| \mu = \mu_0)$$

$$+ p((\overline{x}_{j-7}, \ldots\ldots, \overline{x}_j) \in [LCL, CL] \| \mu = \mu_0)$$

$$+ p((\overline{x}_{j-7}, \ldots\ldots, \overline{x}_j) \in [CL, UCL] \| \mu = \mu_0)$$

$$=2\,\Phi(-k)+2[.5-\Phi(-k)]^{8} \qquad (2.2.2.2)$$

2- The non-detection of an existing shift (type-II error):

This type of error occurs when the production process is out of control but the control procedure shows that it is under control. The probabilities of its occurrences are:

β_1 = p**(** no out-of control signal from the j^{th} sample **,**($j<8$) | process is in state II)

$$\beta_1 = p\left(\bar{x}_j \in [LCL, UCL] \mid \mu \neq \mu_o\right) = \Phi\left(k-\delta\sqrt{n}\right)-\Phi\left(-k-\delta\sqrt{n}\right) \qquad (2.2.2.3)$$

Hence:

$1-\beta_1$ = p (detecting an existed shift from the j^{th} sample, j<8)

$$=1-\Phi\left(k-\delta\sqrt{n}\right)+\Phi\left(-k-\delta\sqrt{n}\right)$$

$$=\Phi\left(-k+\delta\sqrt{n}\right)+\Phi\left(-k-\delta\sqrt{n}\right)$$

$1-\beta_2$ = P **(** detecting a shift by the j^{th} sample **,** for $j \geq 8$ | process in state II**)**

$$1-\beta_2 = \left\{1-\left[\Phi\left(k-\delta\sqrt{n}\right)-\Phi\left(-k-\delta\sqrt{n}\right)\right]\right\}+\left[\Phi\left(-k-\delta\sqrt{n}\right)-\Phi\left(-\delta\sqrt{n}\right)\right]^{8}$$

$$+\left[\Phi\left(-\delta\sqrt{n}\right)-\Phi\left(k-\delta\sqrt{n}\right)\right]^{8}$$

β_2 = p (no out - of - control signal is up to the j^{th} sample inclusive , ($j \geq 8$) | process is in state II **)**

$$= 1-(1-\beta_2)$$

$$\beta_2 = \left\{\Phi\left(k-\delta\sqrt{n}\right)-\Phi\left(-k-\delta\sqrt{n}\right)\right\}-\left[\Phi\left(-k-\delta\sqrt{n}\right)-\Phi\left(-\delta\sqrt{n}\right)\right]^{8}$$

$$- \left[\Phi\left(-\delta\sqrt{n}\right) - \Phi\left(k-\delta\sqrt{n}\right)\right]^8 \qquad (2.2.2.4)$$

Where:

$$\Phi(z) = p(Z \le z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{x^2}{2}} dx$$

The average runs length (ARL):

The average number of samples required for the first warning is called the average run length (ARL) which represents the average number of samples drawn to give the first indication that the process is out of control.

Assuming that D is a random variable representing the number of samples drawn to get the first warning signal, thus

$ARL = E[D \mid \mu]$.

There are two types of the average run length (ARL):

(i) The average run length when the process is in state I (ARL1) represents the average number of samples to be drawn to obtain the first false alarm. The p.m.f of the random variable D in this case is:

p_j = The p. m. f that the 1^{st} false alarm will be obtained at the j^{th} sample for any r.

$$p_j = \begin{cases} (1-\alpha_1)^{r-1}\alpha_1\,, & j < r \\ (1-\alpha_1)^{r-1}(1-\alpha_2)^{j-r}\alpha_2\,, & j \ge r \end{cases}$$

$$ARL1 = E\left[D \mid STATE\ I\right] = \sum_{j=1}^{\infty} jp_j = \sum_{j=1}^{r-1} jp_j + \sum_{j=r}^{\infty} jp_j$$

$$= \frac{1-(1-\alpha_1)^r}{\alpha_1} + \frac{(1-\alpha_1)^{r-1}(1-\alpha_2)}{\alpha_2}$$

For the study (r=8),

$$\text{ARL1}= \frac{1-(1-\alpha_1)^8}{\alpha_1} + \frac{(1-\alpha_1)^7(1-\alpha_2)}{\alpha_2} \qquad (2.2.2.5)$$

(ii) The average run length when the process is in state II (ARL2) which represents the average number of samples drawn to detect the shift for the 1^{st} time.

The p.m.f of the random variable D for this case is

p_j^*= The probability mass function that the 1^{st} detection that the process is out of control will be obtained at the j^{th} sample, for any r is:

$$P_j^* = \begin{cases} \beta_1^{j-1}(1-\beta_1), \; j<r \\ \beta_1^{r-1}\beta_2^{j-r}(1-\beta_2), \; j\geq r \end{cases}$$

$$\text{ARL2=E}\left[D \middle| \text{STATE II}\right] = \sum_{j=1}^{\infty} j p_j^* = \sum_{j=1}^{r-1} j p_j^* + \sum_{j=r}^{\infty} j p_j^*$$

$$= \frac{(1-\beta_1^r)}{(1-\beta_1)} + \frac{\beta_1^{r-1}\beta_2}{(1-\beta_2)}$$

For r=8, $\text{ARL2}= \frac{1-\beta_1^8}{1-\beta_1} + \frac{\beta_1^7\beta_2}{1-\beta_2}$ (2.2.2.6)

2.3-The objective function:

To determine the objective function, it is necessary to find the expected value of samples drawn in the state I and state II and the expected number of false alarms. For that let:

1- S_I be a random variable that represents the number of samples drawn in state I , then

$\{S_I = i\}$ **= {**the transition to state II occurs in the interval (ih ,(i+1) h)}

The probability mass function of S_I is:

$$p(s_I = i) = p(ih < T < (i+1)h) = \int_{ih}^{(i+1)h} \lambda e^{-\lambda t} dt$$

$$= (1 - e^{-\lambda h})(e^{-\lambda h})^i \qquad ,i=0,1,2,3,\ldots\ldots$$

This probability mass function is the probability mass function of a geometric distribution with parameter $(1 - e^{-\lambda h})$ .Thus, the expected value of the number of samples drawn in state I is:

$$E(S_I) = \mu_{S_I} = \frac{e^{-\lambda h}}{(1 - e^{-\lambda h})} = \frac{1}{e^{\lambda h} - 1}$$

2- S_{II} be a random variable that represents the number of samples drawn in state II , then

$\{s_{II} = j\}$ **= {**the shift is detected at the j^{th} sample in state II }

Thus, the p.m.f of S_{II} is p_j^*= {The probability mass function that the 1^{st} detection that the process is out of control will be obtained at the j^{th} sample }

Consequently ,the expended number of samples drawn is state II is:

$$E(S_{II}) = \mu_{S_{II}} = ARL2$$

3- S be a random variable represents the number of samples drawn in a cycle then: $S = S_I + S_{II}$

According to this, the expected value of samples drawn is a cycle is:

$$E(S) = \mu_S = E(S_I + S_{II}) = \mu_{S_I} + \mu_{S_{II}}$$

4-F be a random variable that represents the number of false alarms.

Since false alarms occur only from sampling in state I .

Then , $F|S_I \sim b(S_I\ ,\alpha) \Rightarrow E(F|S_I) = \alpha S_I$

Thus, the average number of false alarms is:

$$\mu_F = E(F) = E[E(F|S_I)] = E(\alpha S_I) = \alpha E(S_I) = \frac{E(S_I)}{ARL1} = \frac{1}{(e^{\lambda h} - 1)ARL1}$$

2.3.1-The economic consequences of charting:

The aim of the control procedure is to improve the quality of the product by keeping the process running in under control state as long as possible. In order to control the process two types of actions could be taken.

1-The control actions:

These actions responsible for detecting of out control conditions, these actions consist mainly from sampling and investigations of false alarms, their economic consequences are the costs of sampling. Suppose that the fixed cost per sample is c_1^* and the cost per unit sampled is c_2^* ,then , the costs of a sample of size n is $c_1^* + c_2^* n$, where $c_1^* \geq 0$ and $c_2^* > 0$.

Thus, the total cost of sampling in a cycle is $C_s = \left(c_1^* + c_2^* n\right)\mu_s$.

Suppose that c_3^* the cost of a false alarm which might include the cost of possible shut down of the process during the investigation of false alarms. then, the overall false alarms costs in a cycle is $C_F = c_3^* \mu_F$.

2-Correction procedures:

These procedures are responsible of a transferring the process from out of control state to a control state. They have two consequences:

- **Costs:**

Assuming that the average correction cost for the whole cycle is c_4^*, which include of the costs of locating and repairing the cause of the out of control and the cost the possible shutdown of the process during the repair.

- **Returns:**

Assuming that $g_1>0$ is the price of a unit produced in state I, and $0< g_2(<g_1)$ is the price of a unit produced in state II. Therefore $[g_1 - g_2]$ is the per unit return from transferring the process from state II to state I.

Since the production rate is ν unit per hour and $\frac{1}{\lambda}$ is the average length of state I.

Therefore, the average return from therefore to state I for a cycle is $(g_1 - g_2)\frac{\nu}{\lambda}$.

Then, the net return for a cycle is $b^* = (g_1 - g_2)\frac{\nu}{\lambda} - c_4^*$.

The average number of units produced in a cycle is $N = \mu_s \nu h$ units, then, the average return from producing these units in state II is $g_2 N = g_2 \mu_s \nu h$.

- **The objective function is formulated as following:**

Assuming that P represents the profit for a cycle, then

$P = b^* - C_S - C_F + g_2 N$

Assuming that L is the a average loss in a cycle, then

$L = -p = C_S + C_F - b^* - g_2 N = \left(c_1^* + c_2^*\right)\mu_s - b^* - g_2 N$

Therefore, the loss per unit produced, as a function of the chart parameters (n, k, h) is:

$$L(n,k,h) = \frac{L}{N} = \frac{L}{\mu_s h\nu}$$

$$L(n,k,h) = \frac{(c_1^* + c_2^* n)\,\mu_s - b^* + c_3^*\,\mu_F}{\mu_s h\nu} - g_2$$

$$= \frac{c_3^*}{h\,\nu}\left[\frac{c_1^* + c_2^* n}{c_3^*} - \frac{b^*}{c_3^* \mu_s} + \frac{\mu_F}{\mu_S}\right] - g_2$$

$$= \frac{c_3^*}{h\nu}\left[\frac{c_1^*}{c_3^*} + \frac{c_2^*}{c_3^*} n - \frac{b}{c_3^* \mu_s} + \frac{\mu_F}{\mu_s}\right] - g_2$$

For simplifying the objective function, we define the following relative economic consequences:

$c_0 = \frac{c_1^*}{c_3^*} \geq 0$ Relative fixed cost of sampling

$c_2 = \frac{c_2^*}{c_3^*} > 0$ Relative variable cost for a sampled unit

$b = \frac{b^*}{c_3^*} \geq 0$ Relative yield from correction

Therefore, the objective function is:

$$L\,(n,k,h) = \frac{c_3^*}{h\,\nu}\left\{c_0 + c_2 n - \frac{b - \mu_F}{\mu_s}\right\} - g_2$$

Since:

$$\mu_F = \frac{1}{(e^{\lambda h} - 1)\,ARL1}, \qquad \mu_S = \frac{1}{e^{\lambda h} - 1} + ARL2$$

Then,

$$L(n,k,h) = \frac{c_3^*}{h\,\nu}\left[c_0 + c_2 n - \frac{b\,(e^{\lambda h} - 1) - \frac{1}{ARL1}}{1 + ARL\,2\,(e^{\lambda h} - 1)}\right] - g_2$$

For more simplification of the objective function add g_2, multiplying it by $\frac{\nu}{\lambda\, c_3^*}$ and let $s = \delta\sqrt{n}$, $x = \lambda\, h$.

Then, the standardized loss function is given below:

$$L_s\,(s,k,x) = \left[L\left(k, \frac{s^2}{\delta^2}, \frac{h}{\lambda}\right) + g_2\right]\frac{\nu}{\lambda\, c_2^*}$$

$$= \frac{1}{x}\left[c_0 + c_2\,\frac{s^2}{\delta^2} - \frac{b(e^x - 1) - \frac{1}{ARL1}}{1 + ARL2(e^x - 1)}\right]$$

$$= \frac{1}{x}\left[c_0 + c_1\, s^2 - \frac{b(e^x - 1) - \frac{1}{ARL1}}{1 + ARL2(e^x - 1)}\right]$$

Where: $c_1 = \frac{c_2}{\delta^2}$,

$$\text{ARL1} = \frac{1-(1-\alpha_1)^8}{\alpha_1} + \frac{(1-\alpha_1)^7(1-\alpha_2)}{\alpha_2}$$

$$\text{ARL 2} = \frac{(1-\beta^8)}{(1-\beta_1)} + \frac{\beta_1^7\beta_2}{(1-\beta_2)}$$

Thus, we have reached the objective function which will be use to determine the optimum values (s^*, k^*, x^*) such that

$$L_s\,(s^*, k^*, x^*) \le L_s(s, k, x) \;\forall\, (s, k, x)$$

Chapter III

Modeling

3.1 Introduction:

The objective of this chapter is to formulate models that can be used to estimate the optimal values of the parameters (n, k, h) for the $\overline{X}-$ chart with run of the 8 points . This procedure requires the minimization of the objective function derived in the last chapter for given values of c_0, c_1 and b. Then, the obtained results are used to build models for the parameters of the chart .

3.2 Minimization procedure:

The optimal values of the parameters (n, k, h) are those value (n^*, k^*, h^*) which minimizes the objective function $L(n, k, h)$. That is ,to find (n^*, k^*, h^*) which satisfies

$$L(n^*, k^*, h^*) \leq L(n,\ k,\ h)\ \forall (n, k, h) \qquad (3.2.1)$$

for different combinations of (c_0, c_1, b).

Equivalently, to determine the value (s, k, x) such that the standardized loss function

$$L_S(s, k, x) = \frac{1}{x}\left[c_0 + c_1 s^2 - \frac{b(e^x - 1) - \frac{1}{ARL1}}{1 + ARL2(e^x - 1)}\right] \qquad (3.2.2)$$

is minimized for each combination of (c_0, c_1, b). That is to find the value (s^*, k^*, h^*) of (s, k, h) such that

$$L_S(s^*, k^*, x^*) \le L_S(s, k, x) \quad \forall \quad (s, k, x) \qquad (3.2.3)$$

for each combination of (c_0, c_1, b).

3.2.1 Numerical minimization:

The objective function (3.2.2) is minimized numerically by the nonlinear programming approach through writing a FORTRAN program using (FORTRAN Power Station 4) for $c_0 = 0$, $c_1 \in [.001, 5.0]$ and $b \in [1.0, 1000]$. A sample of the output $\left(s^*, k^*, x^*\right)$ of the minimization procedure is given in table(3-1) below.

Table (3-1): A sample of the output $\left(s^*, k^*, x^*\right)$ and the value of L_S for given values of b and c_1

b	c_1	s^*	k^*	x^*	L_s
1	0.001	2.89057	4.22332	0.24193	-0.76126
1	0.005	2.28615	3.31135	0.42533	-0.59788
1	0.01	1.95836	2.86637	0.56798	-0.50236
2	0.03	1.45902	2.24355	0.58413	-0.96785
2	0.04	1.25533	2.00705	0.66565	-0.8955
5	0.08	0.91891	1.63253	0.48464	-2.70215
6	0.001	2.9382	4.31449	0.09408	-5.38212
9	0.1	0.83498	1.53607	0.36328	-5.55117
10	0.001	2.94514	4.32796	0.07234	-9.19552
50	0.1	1.08359	1.79279	0.13264	-40.8912
100	0.001	2.96101	4.35889	0.02248	-97.4049

150	0.09	1.19595	1.93177	0.07222	-134.184
150	0.1	1.1315	1.85096	0.07406	-133.695
200	0.001	2.96311	4.36301	0.01586	-196.32
250	0.04	1.62006	2.47766	0.04476	-234.173
250	0.05	1.51539	2.34158	0.04767	-232.919
250	0.06	1.42587	2.22584	0.05008	-231.853
250	0.1	1.14506	1.86794	0.05681	-228.741
300	0.001	2.96404	4.36483	0.01294	-295.488
350	0.1	1.15208	1.87683	0.04777	-324.713
400	0.001	2.96459	4.36591	0.0112	-394.786
450	0.001	2.9648	4.36631	0.01056	-444.468
450	0.005	2.44614	3.57999	0.01719	-440.143
550	0.01	2.19621	3.23931	0.01949	-535.811
550	0.02	1.92544	2.87975	0.02434	-531.591
800	0.1	1.16472	1.89298	0.03131	-761.4
850	0.001	2.96571	4.3681	0.00767	-842.388
850	0.005	2.44794	3.58316	0.01248	-836.428
900	0.1	1.16611	1.89477	0.02949	-859.015
950	0.001	2.96584	4.36836	0.00726	-941.952
1000	0.09	1.22837	1.97404	0.0273	-958.114
1000	0.1	1.16728	1.89627	0.02795	-956.758

3.3 Models building:

Minimizing the objective function at the production line is not convenient, instead it is suitable to have models which can be used to determine the values of each component of the triplet (s^*, k^*, x^*) as dependent variable from the knowledge of c_1 and b as independent variables. It has been found that the rational regression model(which is supplied by sigma-stat package or by Table curve 3D V2) provides the best fit for the data.

Applying this procedure to the output of the minimization of the objective function gives the following models:

1-Model for k^*:

$$\hat{k}^* = \frac{.5579 + .0529\ln b - .4086\ln c_1 - .0058(\ln b)^2 - .1935(\ln c_1)^2 + .1338\ln b\ln c_1}{1 - .1071\ln b + .3366\ln c_1 - .0026(\ln b)^2 - .0012(\ln c_1)^2 + .0113\ln b\ln c_1} \quad (3.3.1)$$

$\overline{R}^2 = .9999,$ the p – value for all coefficients are ≤ 0.0001

2-Model for s^*:

$$\hat{s}^* = \frac{-.9278 + .2648\ln b - 1.2146\ln c_1 - .0153(\ln b)^2 - .3091(\ln c_1)^2 - .1589\ln c_1 \ln b}{1 - .0391\ln b + .263\ln c_1 - .0103(\ln b)^2 - .0359(\ln c_1)^2 + .029\ln b\ln c_1} \quad (3.3.2)$$

$\overline{R}^2 = .9999,$ the p – value for all coefficients are ≤ 0.0001

3-model for x^*:

$$\hat{x}^* = \frac{-.7385 + .158\ln b - .01(\ln b)^2 + .0029(\ln c_1)^2}{1 - .4659\ln b + .4884\ln c_1} \quad (3.3.3)$$

$\overline{R}^2 = .9978,$ the p – value for all coefficients are ≤ 0.0001

These models are applied for some selected values of b and c_1, a sample of estimated values$(\hat{s}^*, \hat{k}^*, \hat{x}^*)$ is given in table (3-2) below.

Table(3-2): The estimated values of the triplet (s^*, k^*, x^*), and the estimated standardized loss function $\hat{L}_S(\hat{s}^*, \hat{k}^*, \hat{x}^*)$ for selected values of b and c_1.

b	c_1	$\hat{s}^*$	$\hat{k}^*$	$\hat{x}^*$	$\hat{L}_s$
1	.001	2.8825	4.217	0.2519	-0.7613
1	.005	2.2636	3.3008	0.4131	-0.5979
1	0.01	1.9445	2.863	0.5413	-0.5024
2	0.03	1.4584	2.2441	0.5771	-0.9679
2	0.04	1.2669	2.0181	0.6741	-0.8955
5	0.08	0.9056	1.6278	0.4995	-2.7022
6	.001	2.9522	4.3285	0.1081	-5.3821
9	0.1	0.8399	1.5325	0.369	-5.5512
10	.001	2.9625	4.3462	0.0832	-9.1955
50	0.1	1.0847	1.7946	0.132	-40.8912
100	.001	2.9728	4.3705	0.018	-97.4049
150	0.09	1.1902	1.926	0.0715	-134.185
150	0.1	1.1272	1.8462	0.0736	-133.695
200	.001	2.9674	4.3645	.00827	-196.32
250	0.04	1.62	2.4723	0.0442	-234.173
250	0.05	1.5131	2.3354	0.0471	-232.919
250	0.06	1.422	2.2195	0.0496	-231.853
250	0.1	1.142	1.8651	0.057	-228.741
300	.001	2.9628	4.359	.00405	-295.488
350	0.1	1.1507	1.8764	0.0486	-324.713
400	.001	2.9591	4.3542	.000162	-394.786
450	.001	2.9575	4.3618	.00177	-444.468
450	.005	2.4513	3.6002	0.0139	-440.143
550	0.01	2.2016	3.2502	0.0179	-535.811
550	0.02	1.9296	2.8812	0.024	-531.591
800	0.1	1.1695	1.9013	0.0344	-761.4
850	.001	2.9475	4.3389	.00275	-842.388
850	.005	2.4463	3.5959	.00903	-836.428
900	0.1	1.172	1.9046	0.0329	-859.015

950	.001	2.9456	4.3363	.00317	-941.952
1000	0.09	1.2319	1.981	0.0309	-958.114
1000	0.1	1.1741	1.9075	0.0318	-956.758

From tables (3-1) and (3-2), there might be some differences between (s^*, k^*, x^*) and $(\hat{s}^*, \hat{k}^*, \hat{x}^*)$ but these differences have no effect on value of the objective function.

That is , $\hat{L}_s(\hat{s}^*, \hat{k}^*, \hat{x}^*) \approx L_s(s^*, k^*, x^*)$

3.4 - Algorithms to determine the values of parameters(n^*, k^*, h^*)

This section gives algorithms to obtain the values of parameters $(\hat{n}^*, \hat{k}^*, \hat{h}^*)$from the knowledge of:

1- The value of the average of in control time $1/\lambda$.

2-The benefit per unit b^* .

3-The variable sampling cost c_2^* .

4-The cost of a false alarm c_3^* .

5-The amount of deviation δ .

6-Determine the values of relative economic consequences:

$$c_1 = \frac{c_2^*}{c_3^* \delta} \quad , \quad b = \frac{b^*}{c_3^*} .$$

7-Applying the following appropriate algorithm to estimate the corresponding parameter.

3.4.1- $\hat{h}^*$ Algorithm:

To determine the value of $\hat{h}^*$:

1-Determine the optimal value of $\hat{x}^*$ from its model (3.3.3).

2-Determine $\hat{h}^*$ from the equation $\hat{h}^* = \hat{x}^* / \lambda$.

This algorithm is applied for some values of c_1, b and λ, a sample of the results is given in table(3-3) below.

Table (3-3) The exact and the approximate optimal sampling interval $(h^*, \hat{h}^*)$ for values of c_1,b and λ.

b	c_1	h^*				$\hat{h}^*$			
		$\lambda=.05$	$\lambda=.5$	$\lambda=1$	$\lambda=3$	$\lambda=.05$	$\lambda=.5$	$\lambda=1$	$\lambda=3$
1	0.001	4.8386	0.48386	0.24193	0.080643	5.038	0.5038	0.2519	0.083967
1	0.005	8.5066	0.85066	0.42533	0.141777	8.262	0.8262	0.4131	0.1377
1	0.01	11.3596	1.13596	0.56798	0.189327	10.826	1.0826	0.5413	0.180433
2	0.03	11.6826	1.16826	0.58413	0.19471	11.542	1.1542	0.5771	0.192367
2	0.04	13.313	1.3313	0.66565	0.221883	13.482	1.3482	0.6741	0.2247
5	0.08	9.6928	0.96928	0.48464	0.161547	9.99	0.999	0.4995	0.1665
6	0.001	1.8816	0.18816	0.09408	0.03136	2.162	0.2162	0.1081	0.036033
9	0.1	7.2656	0.72656	0.36328	0.121093	7.38	0.738	0.369	0.123
10	0.001	1.4468	0.14468	0.07234	0.024113	1.664	0.1664	0.0832	0.027733
50	0.1	2.6528	0.26528	0.13264	0.044213	2.64	0.264	0.132	0.044
100	0.001	0.4496	0.04496	0.02248	0.007493	0.36	0.036	0.018	0.006
150	0.09	1.4444	0.14444	0.07222	0.024073	1.43	0.143	0.0715	0.023833
150	0.1	1.4812	0.14812	0.07406	0.024687	1.472	0.1472	0.0736	0.024533

200	0.001	0.3172	0.03172	0.01586	0.005287	0.1654	0.01654	0.00827	0.002757
250	0.04	0.8952	0.08952	0.04476	0.01492	0.884	0.0884	0.0442	0.014733
250	0.05	0.9534	0.09534	0.04767	0.01589	0.942	0.0942	0.0471	0.0157
250	0.06	1.0016	0.10016	0.05008	0.016693	0.992	0.0992	0.0496	0.016533
250	0.1	1.1362	0.11362	0.05681	0.018937	1.14	0.114	0.057	0.019
300	0.001	0.2588	0.02588	0.01294	0.004313	0.081	0.0081	0.00405	0.00135
350	0.1	0.9554	0.09554	0.04777	0.015923	0.972	0.0972	0.0486	0.0162
400	0.001	0.224	0.0224	0.0112	0.003733	0.00324	0.000324	0.000162	0.000054
450	0.001	0.2112	0.02112	0.01056	0.00352	0.0354	0.00354	0.00177	0.00059
450	0.005	0.3438	0.03438	0.01719	0.00573	0.278	0.0278	0.0139	0.004633
550	0.01	0.3898	0.03898	0.01949	0.006497	0.358	0.0358	0.0179	0.005967
550	0.02	0.4868	0.04868	0.02434	0.008113	0.48	0.048	0.024	0.008
800	0.1	0.6262	0.06262	0.03131	0.010437	0.688	0.0688	0.0344	0.011467
850	0.001	0.1534	0.01534	0.00767	0.002557	0.055	0.0055	0.00275	0.00092
850	0.005	0.2496	0.02496	0.01248	0.00416	0.1806	0.01806	0.00903	0.00301
900	0.1	0.5898	0.05898	0.02949	0.00983	0.658	0.0658	0.0329	0.010967
950	0.001	0.1452	0.01452	0.00726	0.00242	0.0634	0.00634	0.00317	0.00106
1000	0.09	0.546	0.0546	0.0273	0.0091	0.618	0.0618	0.0309	0.0103
1000	0.1	0.559	0.0559	0.02795	0.009317	0.636	0.0636	0.0318	0.0106

3.4.2 $\hat{k}^*$ Algorithm

To determine the value of $\hat{k}^*$:

1- Use the k model (3.3.1) to estimate the values of $\hat{k}^*$

The following table contains a sample of ($\hat{k}^*, k^*$) for given values of b and c_1.

Table (3-4) :The exact and the approximate optimal control limits ($k^*, \hat{k}^*$)for selected values of b and c_1.

b	c_1	k^*	$\hat{k}^*$
1	.001	4.22332	4.217
1	.005	3.31135	3.3008
1	0.01	2.86637	2.863
2	0.03	2.24355	2.2441

2	0.04	2.00705	2.0181
5	0.08	1.63253	1.6278
6	.001	4.31449	4.3285
9	0.1	1.53607	1.5325
10	.001	4.32796	4.3462
50	0.1	1.79279	1.7946
100	.001	4.35889	4.3705
150	0.09	1.93177	1.926
150	0.1	1.85096	1.8462
200	.001	4.36301	4.3645
250	0.04	2.47766	2.4723
250	0.05	2.34158	2.3354
250	0.06	2.22584	2.2195
250	0.1	1.86794	1.8651
300	.001	4.36483	4.359
350	0.1	1.87683	1.8764
400	.001	4.36591	4.3542
450	.001	4.36631	4.3618
450	.005	3.57999	3.6002
550	0.01	3.23931	3.2502
550	0.02	2.87975	2.8812
800	0.1	1.89298	1.9013
850	.001	4.3681	4.3389
850	.005	3.58316	3.5959
900	0.1	1.89477	1.9046
950	.001	4.36836	4.3363
1000	0.09	1.97404	1.981
1000	0.1	1.89627	1.9075

3.4.3- $\hat{n}^*$ Algorithm:

To determine the value of $\hat{n}^*$:

1-Use s-model (3.3.2) to determine $\hat{s}^*$.

2- $\hat{n}^*$ is determined using the equation $\hat{n}^* = \left[\frac{\hat{s}^*}{\delta^2}\right]$.

Table (3-5): A sample of some of the values ($n^*, \hat{n}^*$) for given values of b and c_1 .

b	c_1	n^*			$\hat{n}^*$		
		$\delta = .5$	$\delta = 1.5$	$\delta = 2$	$\delta = .5$	$\delta = 1.5$	$\delta = 2$
1	0.001	33	4	2	33	4	2
1	0.005	21	2	1	21	2	1
1	0.01	15	2	1	15	2	1
2	0.03	9	1	1	9	1	1
2	0.04	6	1	0	6	1	0
5	0.08	3	.4	0	3	0	0
6	0.001	35	4	2	35	4	2
9	0.1	3	0	0	3	0	0
10	0.001	35	4	2	35	4	2
50	0.1	5	1	0	5	1	0
100	0.001	35	4	2	35	4	2
150	0.09	6	1	0	6	1	0
150	0.1	5	1	0	5	1	0
200	0.001	35	4	2	35	4	2
250	0.04	11	1	1	11	1	1
250	0.05	9	1	1	9	1	1
250	0.06	8	1	1	8	1	1
250	0.1	5	1	0	5	1	0
300	0.001	35	4	2	35	4	2
350	0.1	5	1	0	5	1	0
400	0.001	35	4	2	35	4	2
450	0.001	35	4	2	35	4	2
450	0.005	24	3	2	24	3	2
550	0.01	19	2	1	19	2	1
550	0.02	15	2	1	15	2	1
800	0.1	5	1	0	6	1	0
850	0.001	35	4	2	35	4	2
850	0.005	24	3	2	24	3	2
900	0.1	5	1	0	6	1	0
950	0.001	35	4	2	35	4	2
1000	0.09	6	1	0	6	1	0
1000	0.1	6	1	0	6	1	0

Chapter IV

Sensitivity Analysis

4.1- Introduction

This chapter devoted to study the effects of the sample size and the control limits on the ability of the chart to detect shifts, also to discuss the impact of the economic consequences and the other process aspects on the estimated parameters of the chart. Thus, this chapter consists of two sections beside the introductory. Section 4.2 deals with the effects of the sample size and the control limits on the sensitivity of the chart, section 4.3 discusses the impacts of the economic the consequences and the other process aspects on the optimal design of the chart.

4.2-The effects of the sample size and control limits on the sensitivity of the chart:

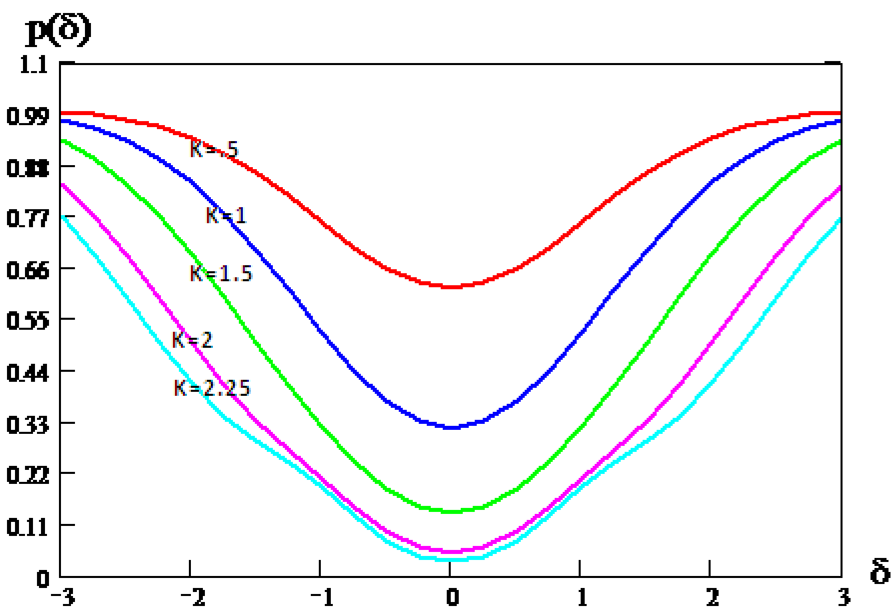
The ability of the chart to detect shifts in the process level can be measured by the power of the chart and the average run lengths (ARLS). Hence, this section is divided into two subsections. Subsection 4.2.1 studies the relations between the chart parameters (n, k) and the power of the chart and subsection 4.2.2 discusses the ARLS. section 4.3 studies the effects of the economic consequences and the other process' aspects on the optimal design of the chart.

4.2.1-The effects of n and k on the power of the chart:

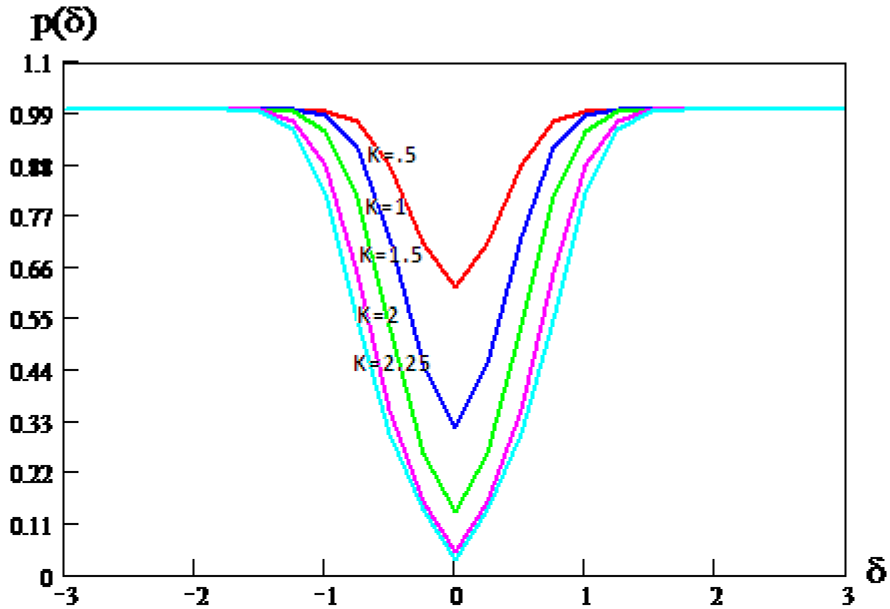
The power of the chart $p(\delta)=1-\beta_2$,where β_2 as shown in equation (2.2.2.4) represents the probability of detecting a given shift size in the process level. This power can be seen as a function of δ,n and k, hence, it is wise to investigate its behavior according to n, k and δ.

I) The effect of the control limits on the power of the chart:

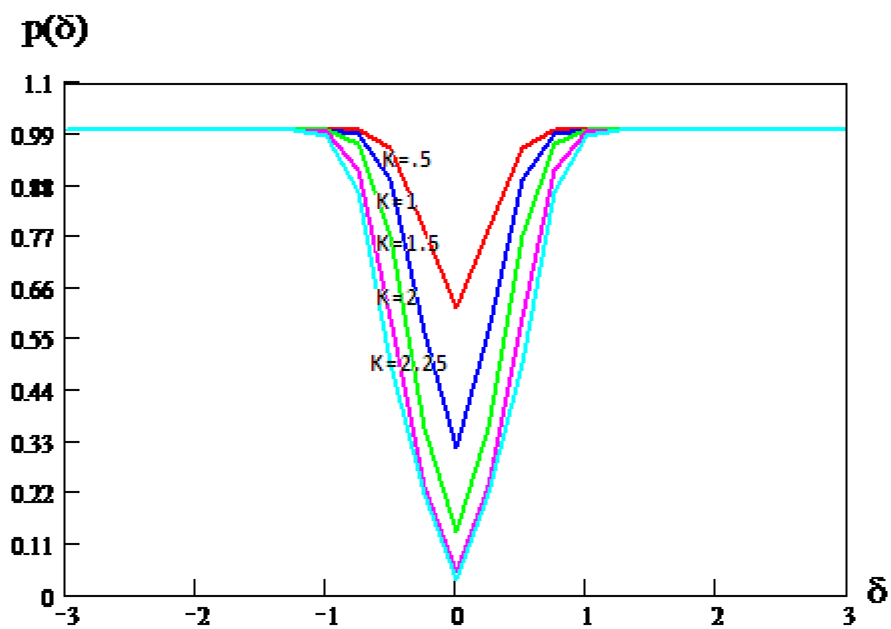
The investigation of the effect of the control limits (k) on the power of the chart is done through the following figures which display the powers of the charts as a function of δ and k for given n's.



Figure(4.1) The power for some values of k , n=1



Figure(4.2) The power for some values of k , n=10



Figure(4.3) The power for some values of k , n=20

Through the earlier figures the following can be noticed:

1)The power of the chart to detect a given shift increases as the width of the control limits decreases.

2)For a given (n,k) , the power of the chart increases as the shift size (δ) increases.

II)The effect of the sample size on the power of the chart:

The effect of the sample size on the power of the chart for a given k can be seen from the following figures.

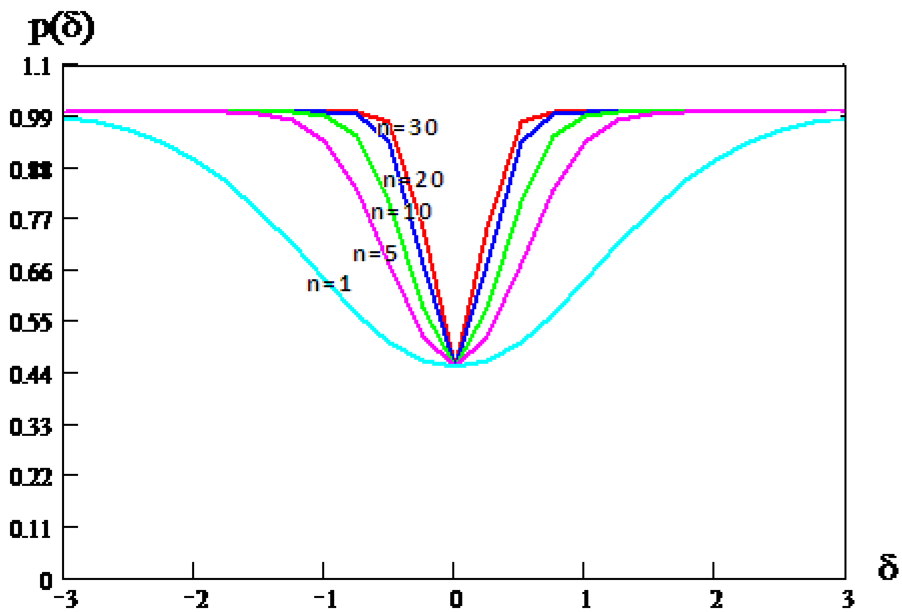
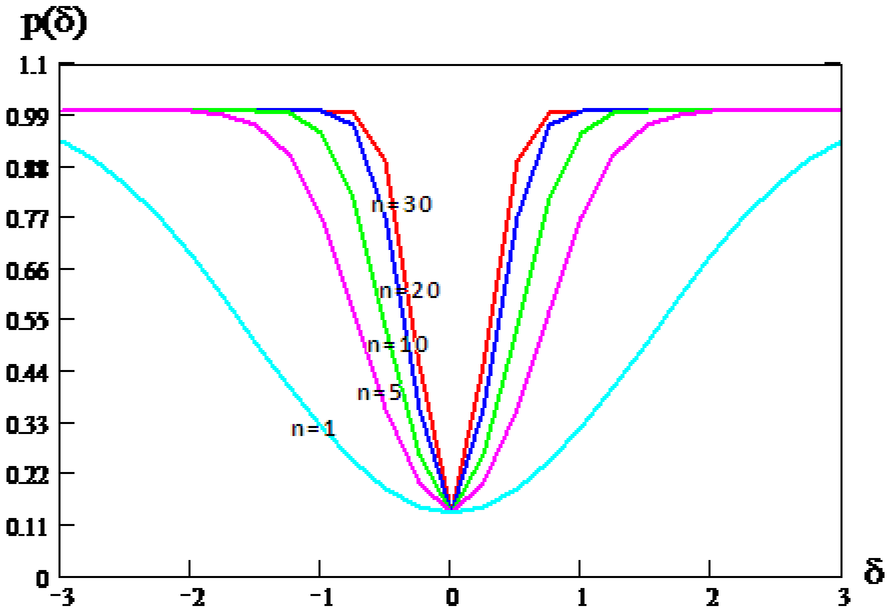


Figure (4.4) The power for some values of n , k = .75



Figure(4.5) The power for some values of n, k = 1.5

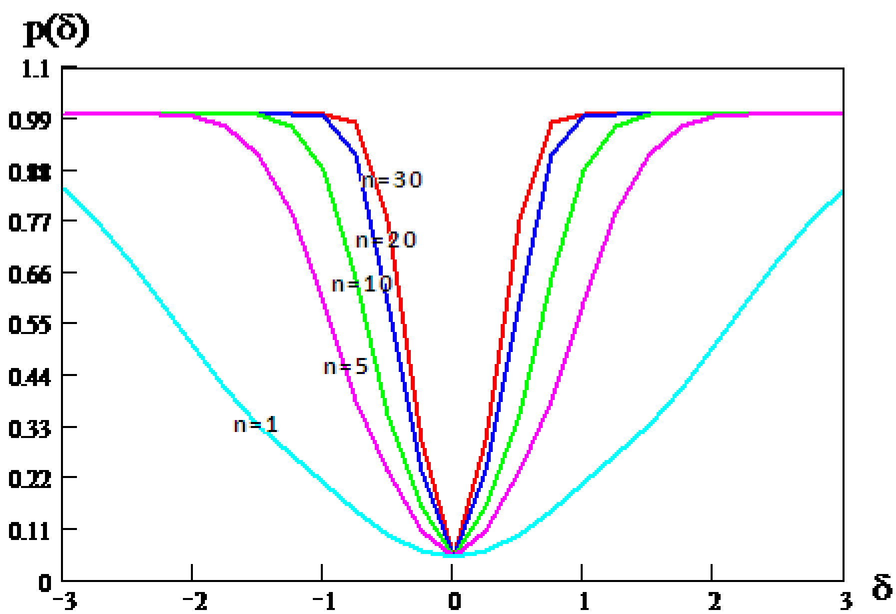


Figure (4.6) The power for some values of n , k = 2

From these figures it can be seen that the power of the chart increases as the sample size increases.

4.2.2-The impact of the chart design on the average runs lengths:

The effect of the sample size n and limit control k on ARL1 equation (2.2.2.5) and ARL2 equation (2.2.2.6) can be seen from the fallowing table:

Table(4-1) Some values ARL1, ARL2 at some values of n, k, δ

		$\bar{x}$ - chart				$\bar{x}$ -chart with runs			
			ARL2				ARL2		
k	n	ARL1	$\delta = .25$	$\delta = .5$	$\delta = .75$	ARL1	$\delta = .25$	$\delta = .5$	$\delta = .75$
3	1	370	281	155	81	97	35	16	11
3	5	370	133	33	11	97	14	8	6
3	10	370	73	13	4	97	10	6	4
3	15	370	47	7	2	97	9	5	2
3	20	370	33	4	2	97	8	4	2
2.5	1	81	66	41	25	51	27	15	10
2.5	5	81	37	12	5	51	13	7	4
2.5	10	81	23	6	2	51	9	5	2
2.5	15	81	16	3	2	51	8	3	2
2.5	20	81	12	3	1	51	7	3	1
2	1	22	19	14	9	20	16	11	7
2	5	22	12	5	3	20	10	5	3
2	10	22	9	3	2	20	7	3	2
2	15	22	7	2	1	20	6	2	1
2	20	22	5	2	1	20	5	2	1

1.5	1	7	7	6	4	7	7	5	4
1.5	5	7	5	3	2	7	5	3	2
1.5	10	7	4	2	1	7	4	2	1
1.5	15	7	3	1	1	7	3	1	1
1.5	20	7	3	1	1	7	3	1	1
1	1	3	3	3	2	3	3	3	2
1	5	3	3	2	1	3	3	2	1
1	10	3	2	1	1	3	2	1	1
1	15	3	2	1	1	3	2	1	1
1	20	3	2	1	1	3	2	1	1
.5	1	2	2	2	1	2	2	2	1
.5	5	2	1	1	1	2	1	1	1
.5	10	2	1	1	1	2	1	1	1
.5	15	2	1	1	1	2	1	1	1
.5	20	2	1	1	1	2	1	1	1

From the previous table we notice:

1-A small sample size leads to an increase in the value of the average number of samples required to obtain a signal and vise verse.

2-A small control limit deceases the average run length and vise verse.

4.3-The effect of the economic consequences on the optimal design of the chart:

This section studies the effects of the economic consequences (b, c_1), on the estimated parameters of the chart. The following table contains a sample of $\left(\hat{n}^*, \hat{k}^*, \hat{h}^*\right)$ for selected values of $b, c_1, \delta = .5, \lambda = .5$.

Table (4-2):Some values of $\hat{n}^*, \hat{k}^*$ and $\hat{h}^*$ for same values of b and c_1

b	c_1	$\hat{n}^*$	$\hat{k}^*$	$\hat{h}^*$
1	0.001	71	2.8825	0.157294
1	0.005	44	2.2636	0.346086
1	0.01	33	1.9445	0.547904
4	0.001	74	2.9413	0.0869
4	0.005	48	2.3698	0.167947
4	0.01	37	2.0846	0.242349
8	0.001	75	2.9584	0.063413
8	0.005	49	2.4029	0.117025
8	0.01	39	2.1275	0.164888
50	0.001	76	2.9749	0.024471
50	0.005	51	2.4477	0.041672
50	0.01	41	2.1878	0.057775
100	0.001	76	2.9728	0.016012
100	0.005	52	2.4532	0.026741
100	0.01	42	2.1971	0.037686
200	0.001	76	2.9674	0.009975
200	0.005	52	2.4543	0.016542
200	0.01	42	2.2016	0.024164
350	0.001	76	2.9609	0.006706
350	0.005	52	2.4527	0.01109
350	0.01	42	2.2024	0.016981
600	0.001	76	2.9532	0.004686
600	0.005	52	2.4493	0.007751
600	0.01	42	2.2013	0.012629

800	0.001	75	2.9486	0.004023
800	0.005	52	2.4469	0.006666
800	0.01	42	2.2001	0.011181
850	0.001	75	2.9475	0.003919
850	0.005	52	2.4463	0.006494
850	0.01	42	2.1998	0.01091
900	0.001	75	2.9466	0.003831
900	0.005	52	2.4458	0.006348
900	0.01	42	2.1995	0.01073
1000	0.001	75	2.9447	0.003695
1000	0.005	52	2.4448	0.006122
1000	0.01	42	2.1989	0.010369

From this table, the following results are obtained:

I) Increasing the per unit sampling cost(c_1) decreases both the optimum sample size ($\hat{n}^*$) and the optimum control limits ($\hat{k}^*$) but increases the optimum sampling interval($\hat{h}^*$).

II) Increasing the benefit per renewal (b) produces some increase in sample size and control limit but reduces the sampling interval.

Chapter V

Conclusion and Future studies

5.1-Conclusion

The goal of this study is to increase the sensitivity of $\overline{X}$-chart to detect small shifts in the process level by using a run of 8 points and to produce models to approximate the optimal design of the chart following the economical approach.

The objectives have been achieved though the discussion of the methodology, the formulation and the minimization of the objective function, and building models and algorithms to determine the economical approximate optimal design of $\overline{X}$-chart with a run of 8 points.

From the analysis of the models, algorithms, charts, and numerical results the following general conclusions are drawn.

1.The implementation of the rule of a run of 8 points increases the sensitivity of the chart.

2.A quick detection of a shift can be obtained by larger samples, shorter sampling interval and small control limits.

3.The larger the shift size the easer to be detected.

4.Changing the economic consequences and the other process aspects produces changes in the chart parameters in the following manner:

a)Changing the variable sampling cost produces a change in the same direction of the sampling interval, and a change in the opposite direction for the sample size and control limits.

b)Changing the benefit per renewal produces a change in the same direction for the sample size and the control limits, and a change in the opposite direction for the sampling interval.

c)Changing the intensity parameter(λ) produces a change in the opposite direction for the sampling interval.

d)Changing the magnitude of the shift(δ) produces a change in the opposite direction for the sample size(n).

5.2-proposed studies:

This study is based on several assumptions for the process and economic consequences for the control procedure, using different assumptions results in topics to be investigated such as:

1.The optimal economic design of $\overline{X}$-chart with a run of r points.

2.The optimal economic design of $\overline{X}$-chart with a run of r (or 8) points when $\overline{X}$ does not follow a normal distribution.

Dedication

To My Children: Moheamen,

Moumen and Loujjee

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I would like to thank Allah without whom this thesis could not be possible. I also would like to thank my supervisor, *Dr. Ali Aldeeb* for his unlimited support, guidance and patience during the whole course of this study. I am grateful to all staff members and colleagues at the Department of Statistics for their useful discussion, friendship and support.

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Finally time has come to thank my husband *Tarek* and my parents for their help, patience and support during the various stages of my study.

Abstract

Quality took a wide interest by many researchers due its importance and ability for achieving the goals of different organizations. However, the quality of production processes affected by many factors which must be monitored. Statistical control charts are considered to be among the best control methods to control the quality of products during the flow of production. $\overline{X}$*-chart is the most applicable chart. It is a diagram consists of three lines: the control line (CL=*μ_0*) indicates the target quality level ,upper control limit(UCL=*$\mu_0 + k\sigma_{\overline{x}}$ *)and lower control limit(LCL=*$\mu_0 - k\sigma_{\overline{x}}$*). For this chart to be used, it must be designed first which means the determination of: sample size(n), the width of control limits(k)and length of the Sampling interval (h). Despite of its simplicity,* $\overline{X}$*-chart is insensitive to detect small deviation in the level of quality characteristic of interest. Many methods were recommended to increase the sensitivity to the chart for small deviation, one of them is a follow 8 points above and below the central line and within the control limits, this approach results an* $\overline{X}$*-chart with run of 8 points. This thesis aims to design the* $\overline{X}$*-chart with run of 8 points, and then to build models to estimate the parameters (n,k,h) of the chart. The objectives are achieved through the:*

1-formulation of the function which relates the aspects of the production process, the characteristics of the chart and the control procedure with its economic consequences.

2-determine the values of the parameters (n,k,h) which minimize the objective function.

3-utilization of the obtained results to form mathematical models which can be used to determine the values of the chart parameters from the economic consequences.

ملخص الدراسة

لقد استأثرت الجودة باهتمام واسع لدى العديد من الباحثين لأنها أصبحت القوة الأكثر حسما نحو تحقيق أهداف المنظمات المختلفة سواء الإنتاجية منها أو الخدمية ،غير أن جودة العمليات الصناعية تتأثر بالعديد من العوامل مما يتطلب مراقبتها. تعد خرائط المراقبة الإحصائية من أهم وأكثر الوسائل الرقابية التي يتم اعتمادها للسيطرة على جودة المنتجات أثناء تدفق الإنتاج. تعتبر خريطة $\overline{X}$ من أكثر الخرائط استخداما للسيطرة على العمليات الإنتاجية، والتي تمثل بيانيا بمخطط يحتوي على ثلاث خطوط هي :الخط المركزي($CL=\mu_0$) وهو مستوى الجودة المستهدف، الحد الأعلى لرقابة الجودة ($UCL=\mu_0 + k\sigma_{\overline{x}}$)، وحدها الادنى ($LCL=\mu_0 - k\sigma_{\overline{x}}$). لإستخدام هذه الخريطة نحتاج لتصميم والذي يقصد به تحديد معالمها:(n)حجم العينة،وبعدا حدا الرقابة(k)،وطول فترة المعاينة(h) . على الرغم من بساطة مخطط $\overline{X}$ إلا أنه يعاب عليه عدم حساسيته للانحرافات البسيطة في مستوى جودة العملية الإنتاجية،لذلك اعتمدت عدة أساليب لزيادة حساسيته منها أسلوب تتابع 8 نقاط فوق وتحت الخط المركزي وضمن حدى السيطرة، وبالتالي نحصل على مخطط $\overline{X}$ بتتابع 8 نقاط،والذي معالمه (n,k,h). إن هذه الدراسة تهدف لتصميم مخطط $\overline{X}$ بتتابع 8 نقاط ثم إلى صياغة نماذج يمكن استخدامها لتقدير معالمه عند مختلف التبعات الاقتصادية لعملية الرقابة وذلك من خلال:

1- تحديد دالة الهدف التي تربط العملية الإنتاجية وسمات خريطة وخواصها والتبعات الاقتصادية الناتجة عن أسلوب الرقابة..
2- تحديد قيم المعالم (n,k,h) التي تجعل دالة الهدف أقل ما يمكن.
3- استخدام النتائج المتحصل عليها لصياغة نماذج رياضية تستخدم لتحديد قيم المعالم من قيم المتغيرات الاقتصادية المعتمدة .

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