

## Localization accuracy improving in wireless networks Mathematically

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**Abstract:** As the demand for indoor Location-Based Services (LBSs) increases, there is an increasing interest in improving an accurate indoor positioning and tracking system on moving user nodes.

One of the most important problems in locating objects in indoor environments is the location accuracy. A location can be calculated by collecting and analyzing data on signal strength, time of arrival, time difference of arrival, or angle of arrival. These data are subject to measurement noise and they are sensitive to the medium circumstances. These circumstances produce location error and then the estimation will be inaccurate. There are a lot of techniques to improve the location estimation, which it was received by any existing localization algorithm; one of these approaches is the approach which uses mathematics for accuracy improving. This paper submits a mathematical technique to improve the positioning accuracy using distance and speed factors. This technique uses the relationship of the distance between mobile objects moving together, and the relationship of the distance between locations of the moving object at different times.

**Key Words:** Accuracy improving. Localization, Positioning system, wireless networks.

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### I. Introduction

Wireless communications have become a very important part of everyday life. New applications emerge every day that requires higher processing power from hardware and software. Localization is one of the most crucial issues for many such applications. It is a challenge for researchers to design a localization technique that is not only simple and cheap, but also accurate and robust [9].

There are many techniques to measuring the distance between two or more sensors, where each sensor has to be putted with a ranging device. In wireless networks, the received signal strength (RSS) is a widely used for localization [1]. The RSS approach uses the relation between the loss in the transmitted signal strength and the traveling distance of the signal. Therefore, the strength of the received signal from a known reference node location can be used to estimate the distance between the reference node and the user node. Furthermore, the received signal strength from at least three reference nodes can be used to estimate the 3-D position of a user node. The RSS technique is a member of family of trilateration-based techniques for positioning.

In this kind of techniques [2], if the locations of at least three reference nodes are known, then the coordinates of a user node can be estimated from the measured range between the user node and the three reference nodes. However, these kinds of measurements usually produce several different types of errors, and precise position of user node is hard to be realized. In practice, the user node locations are estimated by least square techniques, which are used to minimize the mean squared errors of the measurements between the user node and the reference nodes [7]. In wireless networks, a multi-literation technique is used, where a user node can estimate its position after measuring the distance from at least three reference nodes. Unfortunately, not each user node can directly measure the distance between itself and the reference nodes. Therefore, the core of any positioning algorithms based on multi-literation is how to obtain precise distance estimates between reference and user nodes [12].

Some localization techniques use the distance and speed relationship of the object being moved. The distance relationship of nodes is defined as relative locations to each other within one group, while the speed relationship of objects is defined as one user node locations at different times [3].

In this paper, this approach is discussed and used to achieve better accuracy than an initial location estimate.

### II. Improving the Accuracy using spatial-temporal relationships of user nodes

The spatial relationship is defined as a relationship between a group of user nodes move together from a location to another and kept their configuration during movement. And the temporal relationship is defined as a relationship of one user node moves from one position to another with a constant direction and speed.

The spatial-temporal relationship is another method using to enhance the accuracy of the position of user node mathematically by using spatial relationship, temporal relationship, and by combining the two relationships. We define user nodes actual locations as  $(x, y)$ , the position that estimated from some known algorithm as  $(x^*, y^*)$ , and the position after adjusting the estimation by applying some methods as  $(x', y')$ .

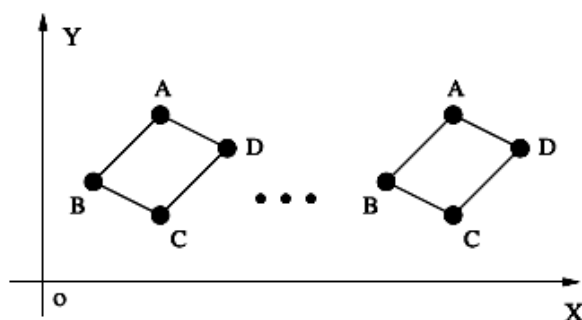


Figure (1) the four user nodes keep their configuration during tracking.

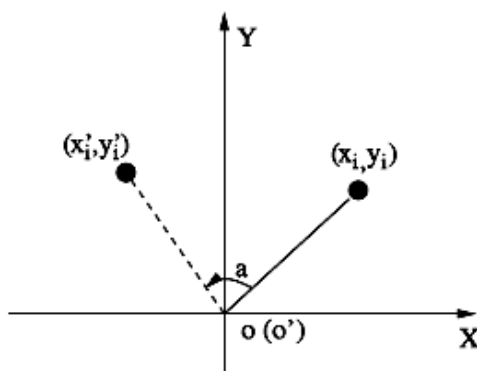


Figure (2) a user node's location relative to the centroid of the group before and after movement.

### III. Improving the localization accuracy by using spatial relationships

Using the space or distance relationship between user nodes in the accuracy enhancement of user node location can be discussed as:

Let  $M$  user nodes  $n_1, n_2, \dots, n_M$  are moving within a building and they did not make any change in either their relative locations or angles as shown in figure (1) [4]. Also let four user nodes (A, B, C, and D) stay together and they do not change their configuration. At a time  $t$ , we can calculate the estimated positions of those user nodes by a localization technique  $(x_{n1}^* - y_{n1}^*), (x_{n2}^* - y_{n2}^*), \dots, (x_{nM}^* - y_{nM}^*)$ .

Let initial relative position of user node  $n_i (1 \leq i \leq M)$  to its group is centroid  $o$  is  $(x_i, y_i)$ . When the group of user node between the location of user node and centroid  $o'$  will be  $(x'_i, y'_i)$ . If  $o'$  transposed to  $o$  (as figure 2) [4], we can express the relation between  $(x_i, y_i)$  and  $(x'_i, y'_i)$  can be expressed as:

$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} \quad (1)$$

Where,  $a$  is an angle after the group moves to a certain location, and  $s$  is an adjustment parameter.

Improvement of the localization is done by minimizing the error value between the estimated locations of user nodes  $(x_i^*, y_i^*) (1 \leq i \leq M)$  which are calculated by a localization algorithm, and adjusted locations  $(x'_i, y'_i) (1 \leq i \leq M)$ , then, the following function must be minimized:

$$f(a, s) = \sum_{i=1}^M \left[ (x'_i - x_i^*)^2 + (y'_i - y_i^*)^2 \right] \quad (2)$$

Now, exchange  $x'_i$  and  $y'$  by their values in (1):

$$f(a, s) = \sum_{i=1}^M \left[ (sx_i \cos a - sy_i \sin a - x_i^*)^2 + (sx_i \sin a + sy_i \cos a - y_i^*)^2 \right]$$

The partial derivatives of  $a$  and  $s$  should be equal to 0:

$$\begin{cases} \frac{\partial f(a, s)}{\partial s} = 0, \\ \frac{\partial f(a, s)}{\partial a} = 0. \end{cases}$$

Then, the value of  $a$  and  $s$  is as follows:

$$\begin{cases} a = \arctan \frac{\sum_{i=1}^M (x_i y_i^* - x_i^* y_i)}{\sum_{i=1}^M (x_i x_i^* + y_i y_i^*)}, \\ s = \frac{\sum_{i=1}^M [(x_i x_i^* + y_i y_i^*) \cos a + (x_i y_i^* - x_i^* y_i \sin a)]}{\sum_{i=1}^M (x_i^2 + y_i^2)}. \end{cases} \quad (3)$$

When  $a$  and  $s$ , are known, we can now calculate of each user node depending on the equation (2), also we can adjust the location of user node to:

$$\begin{bmatrix} x'_{ui} \\ y'_{ui} \end{bmatrix} = \begin{bmatrix} o_x^* \\ o_y^* \end{bmatrix} + \begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} o_x^* \\ o_y^* \end{bmatrix} + s \begin{bmatrix} x_i \cos a - y_i \sin a \\ x_i \sin a + y_i \cos a \end{bmatrix}. \quad (4)$$

#### IV. Improving the localization accuracy by using temporal relationships

Enhancing the accuracy can also be done by monitoring a single user node over the line. Let a user node moves inside the building with a constant speed and direction, and the positions of that user node is known by a localization algorithm from time  $I$  to time  $T$  are  $(x_1^*, y_1^*), (x_2^*, y_2^*), \dots, (x_T^*, y_T^*)$ . Also suppose that, the time 1 is not a start time of movement of user node [11]. The problem is to enhance the accuracy of these estimated locations to make them near to the actual locations. Two techniques under control of two conditions will be done:

- 1- The speed and direction are known.
- 2- The speed and direction are unknown.

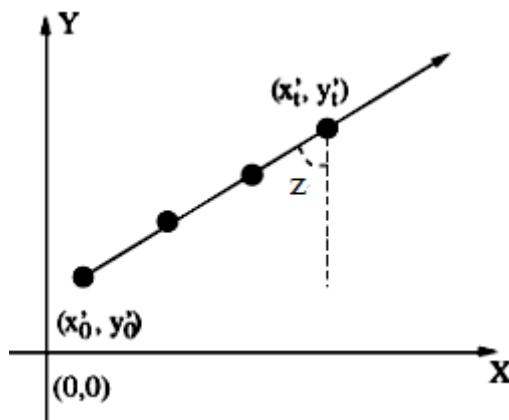


Figure (3) A target's trajectory over a sensor field.

##### First Case: Speed and direction are known

let  $v$  the speed of user node and  $Z$  is the direction of it, as shown in Figure (3) [4], and  $(x'_o, y'_o)$  is the start location, and then the estimated position of this user node at time  $t(1 \leq t \leq T)$  is:

$$\begin{cases} x'_t = x'_o + tv \sin( Z) \\ y'_t = y'_o + tv \cos( Z). \end{cases} \quad (5)$$

Adjusting position to enhance the accuracy will be done by decreasing the error between the estimated position and the actual position, so, the following function must be minimized:

$$f(x'_o, y'_o) = \sum_{t=1}^T \left[ (x'_t - x_t^*)^2 + (y'_t - y_t^*)^2 \right] \quad (6)$$

Exchange  $x'_t, y'_t$  with their values in (5)

$$f(x'_o, y'_o) = \sum \left[ (x'_o + tv \sin( Z) - x_t^*)^2 + (y'_o + tv \cos( Z) - y_t^*)^2 \right]$$

Because the derivatives of  $x'_o, y'_o$  should be equal to zero, then  $x'_o$  and  $y'_o$  can be found as:

$$\begin{cases} x'_o = \frac{1}{T} \sum_{t=1}^T x_t^* - \frac{T+1}{2} v \sin( Z), \\ y'_o = \frac{1}{T} \sum_{t=1}^T y_t^* - \frac{T+1}{2} v \cos( Z) \end{cases} \quad (7)$$

The estimated position  $(x_t^*, y_t^*)$  can be corrected to  $(x'_t, y'_t)$  as follows:

$$\begin{bmatrix} x'_t \\ y'_t \end{bmatrix} = \begin{bmatrix} x'_o + tv \sin( Z) \\ y'_o + tv \cos( Z) \end{bmatrix}. \quad (8)$$

### Second case: Speed and direction are unknown

Let  $v_x$  is the speed of user node in direction  $x$ , and  $v_y$  is the speed in the direction  $y$ , and  $(x'_o, y'_o)$  is the starting position which is known, then the estimated position of this user node at time  $t(1 \leq t \leq T)$  is:

$$\begin{cases} x'_t = x'_o + tv_x \\ y'_t = y'_o + tv_y \end{cases} \quad (9)$$

Adjusting position to enhance the accuracy can be done by decreasing the error between the estimated position and the actual position, so, the following function must be minimized:

$$f(v_x, v_y, x'_o, y'_o) = \sum_{t=1}^T \left[ (x'_t - x_t^*)^2 + (y'_t - y_t^*)^2 \right] \quad (10)$$

Exchange  $x'_t, y'_t$  with their values in (9):

$$f(v_x, v_y, x'_o, y'_o) = \sum_{t=1}^T \left[ (x'_o + tv_x - x_t^*)^2 + (y'_o + tv_y - y_t^*)^2 \right]$$

The derivatives of  $v_x, v_y, x'_o, y'_o$  should be equal to zero:

$$\begin{cases} \frac{\partial f(v_x, v_y, x'_o, y'_o)}{\partial v_x} = 0, \\ \frac{\partial f(v_x, v_y, x'_o, y'_o)}{\partial v_y} = 0, \\ \frac{\partial f(v_x, v_y, x'_o, y'_o)}{\partial x'_o} = 0, \\ \frac{\partial f(v_x, v_y, x'_o, y'_o)}{\partial y'_o} = 0. \end{cases} \quad (11)$$

Each estimated location  $(x_i^*, y_i^*)$  can be adjusted to  $(x_i', y_i')$  as follows:

$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} x_0' + tv_x \\ y_0' + tv_y \end{bmatrix}. \quad (12)$$

### V. Improving the localization accuracy by combining the spatial and temporal relationships

In this section, space and speed relationship between estimated locations and actual location of the user nodes will be combined together in two different ways:

- (1) Distance first and then speed.
- (2) Speed first and then distance.

#### 1. Spatial first, temporal next

In the first case of the combination, estimated position of all user nodes should be known over previous T time locations. To apply the improvement, first use (4) to adjust their locations using as a group using their spatial relationships. Then, use (8) to adjust each user node using its temporal relationship if its direction and speed are known, if not, use (12).

#### 2. Temporal first, spatial next

In the second case of the combination, estimated position of all user nodes should be known over previous T time locations. To apply the improvement, use (8) to adjust each user node using its temporal relationship if its direction and speed are known, if not, use (12). Then, use (4) to adjust their locations using as a group using their spatial relationships.

## VI. Simulations

Besides the theoretical proofs, we also conducted simulations to see if these techniques can further enhance the accuracy of estimated location using a simulator built in the Matlab. We can start with location estimates from some localization algorithm that needs precision. Here, we chose tracking by signal strength, as our algorithms.

Signal-strength localization has errors due to noise and due the presence of obstructions between reference and user nodes in building because. Location estimation based on signal strength is usually not precise. Our experiments are done to prove how these methods can enhance the initial accuracy.

### Simulation for tracking by signal strength

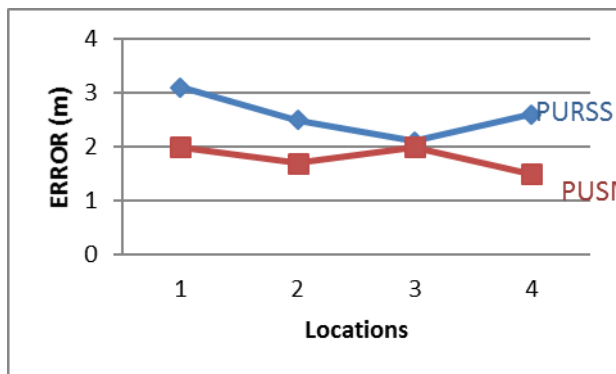
In the implementation step, we will call the process which is using signal strength (PURSS), and it will be compared with all other processes which we will call them as the following:

- Process using the spatial method (PUSM).
- Process using the temporal method when speed and direction are known (PUTM1).
- Process using the temporal method when speed and direction are not known (PUTM2).
- Process using the spatial method first and then the temporal method next (PUSTM).
- Process using the temporal method first and then the spatial method next (PUTSM).

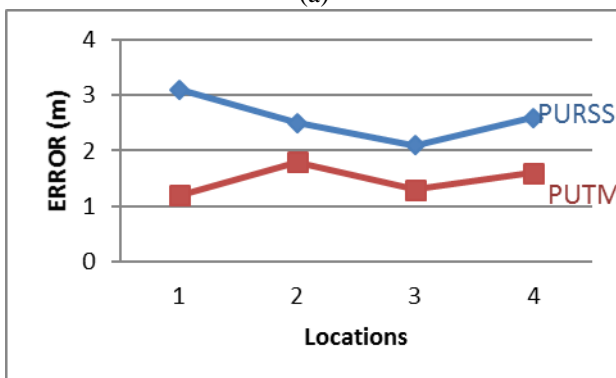
Many experiments are done and implemented using MATLAB software package to show the enhancement of localization accuracy, all experiments are done at a hall with a size of 20m length, and 15m width.

For our simulation, we put a group of 4 nodes. Signals are traveling in a direction with a speed across the hall. As it traveled, we calculated signal strengths received by the user nodes. The time starts from 1 and the interval between time points is 2 second. Then, by considering the space and time relationships of user nodes, we apply AUSM, AUTM1, AUTM2, AUSTM, and AUTSM to adjust the locations of the user nodes. Figure (4 (a) – (e)) show the tracking simulator with a group of four user nodes.

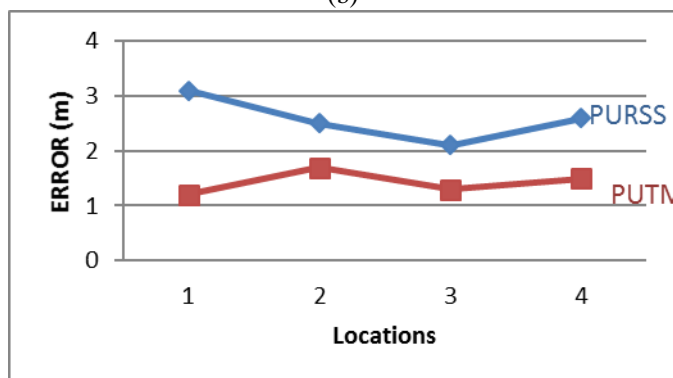
Figure (4) proved the performance of these methods. The processes enhance the localization accuracy of the AURSS algorithm. The figures show that AUTM1 performs better than AUTM2. This is because, if more parameters are known, the results can be more accurate. Similarly, the combination of the spatial and temporal relationships is better than each one alone. However, there was not much difference between AUSTM and AUTSM. This means that whether the locations are adjusted first by the spatial relationships or by the temporal relationships does not matter much. Also, the signal variance has little effect on the tracking error.



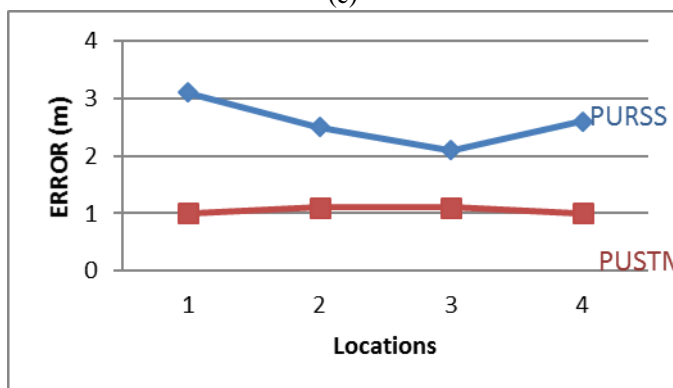
(a)



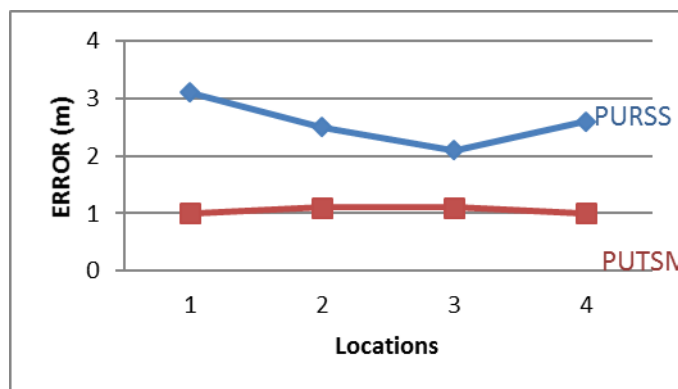
(b)



(c)



(d)



(e)

Figure (4) comparing of algorithms

### VII. Conclusion

Currently there is no indoor localization system that can provide an accurate position estimation for all environments. This paper provides many experiments that obtained the estimated locations for moving objects. A mathematical method is then presented to improve the location of the object using two mathematical models. Also, the paper provides a study on the spatial and temporal relationship of user nodes.

Many Experiments were done, the results of these experiments were improved using spatial-temporal relationships technique.

We showed the difference when we use one technique, and when we combine techniques together.

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