A Recent Treatment for M/D/1 queueing Model with Encouraged Arrivals

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Abstract:

This paper aims to derive the analytical solution of the truncated single-channel queue: M/D/1 at the steady-state by adding the concept of encouraging arrivals. Using the iterative method and the probability generating function method obtain the probability that there are n customers in the system, the possibility of a vacuous system, and some performance effectiveness measures. Some numerical values are given for the additional concept to show the effect of the (arrival rate, encouraging arrivals) on the model so that the companies implementing this model work to reduce the harmful ones.

Keywords: generating function, iterative, queueing, encouraging arrivals, steady-stet.

معالجة حديثة لنموذج انتظار M / D / 1 مع تشجيع الوصول

الملخص

تهدف هذه الورقة لاشتقاق حل في حالة الثبات لنموذج الطابور M/D/1 المضاف له مفهوم تشجيع الوافدين. يتم الحصول على احتمال وجود n من العملاء في النظام، احتمال وجود النظام فارغ, مقاييس فاعلية النظام. اخيراً، يتم إعطاء بعض القيم العددية للمفهوم المضاف لإظهار تأثير مفهومي معدل الوصول تشجيع الوافدين على النموذج لكي تعمل الشركات المنفذة لهذا النموذج على التقليل من الضار منهما.

الكلمات المفتاحية: الدالة المولدة ، التكرارية الطابور ، تشجيع الوافدين ، حالة الثبات.

1.Introduction

In the labor market, companies seek to profit by encouraging arrivals to access their system, through discounts on their products and thus encouraging customers to enter the system. The encouraged arrivals process results in queues on the service channel. For this, it is necessary to work on building a model for the queue, which is under the influence of encouraging arrivals access to the system, in order to anticipate the number of customers in the queue, the number of customers in the system, and the time that the customer spends in the system. The process of encouraging arrivals is a recently studied phenomenon. In 2017, Some and Seth [7] studied the M/M/1/N queue with the impact of encouraging arrivals. Also in 2018, Some and Seth [8] studied the effect of encouraging arrivals on the M/M/1/C queue.

Sometimes the service time is fixed is specified, so we have a queue with one service channel with service time are deterministic. This type of queue has not been well researched because it is difficult to derive its Poisson probabilities. In 2000 Brun and Garcia [1] studied the M/D/1 column by deriving Poisson probability. also, a lesson Nakagawa [2] expands the fixed chain of possibilities for the M/D/1 queue. In 2015, Prasad and Usha [4] compared queuing M/M/1 to M/D/1 in terms of the effectiveness of its application to vehicular traffic in the Kanyakumari district. Hussain et al. [5] studied a simulation of the M/D/1 queue. In 2016, Beak et al. [6] studied the M/D/1 queue in the case of time dependence. Finally, Kotb and Akhdar [9] studied the effect of abstinence on the M/D/1 queue.

This article, suggests deriving the analytical solution for queueing system M/D/1at the steady-state by adding the concepts of encouraging arrivals. The express probability that there are n customers in the system, the probability of empty system obtained using the iterative method, and the probability generating function. Some measures of effectiveness. Finally, a simulation study has been considered to illustrate the numerical application for the model.

2. Basic Notations and Assumptions

To construct the system of this article, we define the following parameters:

$\lambda =$ Mean arriving rate.

$\mu =$ Mean service rate.

$n =$ Number of customers in the system.

$\rho = \lambda D =$ Utilization factor.

$L =$ Expected number of customers in the system.

$L_q =$ Expected number of clientele waiting to be served.

$W =$ Expected waiting time in the system.

$W_q =$ Expected waiting time in the queue.

$\eta =$ Represents the percentage increase in the number of customers computed from past or observed data.

$P(z) =$ The Probability generating func-tion.

$p_n =$ The steady-state possibility that there are n clientele in the system.

$p_0 =$ The steady-state possibility that there are no clientele in the system.

$D =$ The fixed time of service between each customer and the other.

The assumption of this model has listed Clients reach the server one by one according to the Poisson process at a rate

$\lambda(1+\eta)$.

3.Model formulation and analysis model

Due to the lack of a Poisson condition for the server (the service has no distribution), so there is no condition Application differential- difference equations. Therefore, it is necessary to resort to another method to find p_n. This method was represented by using the iterative method and the probability generating function, following as:

$$p_0 = e^{-\lambda(1+\eta)} p_0 + e^{-\lambda(1+\eta)} p_1 \quad (1)$$

$$p_1 = \lambda(1+\eta) e^{-\lambda(1+\eta)} p_0 + \lambda(1+\eta) e^{-\lambda(1+\eta)} p_1 + e^{-\lambda(1+\eta)} p_2 \quad (2)$$

$$p_2 = \frac{[\lambda(1+\eta)]^2 e^{-\lambda(1+\eta)}}{2!} p_0 + \frac{[\lambda(1+\eta)]^2 e^{-\lambda(1+\eta)}}{2!} p_1$$

$$+ \frac{[\lambda(1+\eta)] e^{-\lambda(1+\eta)}}{1!} p_2 + \frac{e^{-\lambda(1+\eta)}}{0!} p_3 \qquad (3)$$

$$p_3 = \frac{[\lambda(1+\eta)]^3 e^{-\lambda(1+\eta)}}{3!} p_0 + \frac{[\lambda(1+\eta)]^3 e^{-\lambda(1+\eta)}}{3!} p_1$$

$$+ \frac{[\lambda(1+\eta)]^2 e^{-\lambda(1+\eta)}}{2!} p_2 + \frac{\lambda(1+\eta)\, e^{-\lambda(1+\eta)}}{1!} p_3$$

$$+ \frac{e^{-\lambda(1+\eta)}}{0!} p_4 \qquad (4)$$

$$\begin{matrix} . & . & . \\ . & . & . \\ . & . & . \end{matrix}$$

$$p_n = \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)]^n}{n!} (p_0 + p_1)$$

$$+ \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)]^{n-1}}{(n-1)!} p_2 + \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)]^{n-2}}{(n-2)!} p_3$$

$$+ \ldots + \frac{e^{-\lambda(1+\eta)}}{0!} p_{n+1} \qquad (5)$$

Thus

$$p_n = \sum_{i=1}^{n} \frac{e^{-\lambda(1+\eta)}\lambda^i}{i!} p_{n-i+1} + \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)]^n}{n!} p_0$$

$$+ e^{-\lambda(1+\eta)} p_{n+1}, \qquad n \geq 1 \qquad \mathbf{(6)}$$

To be finding explicit p_0 in $\lambda(1+\eta)$, we use the probability generating function $P(z)$ whereas:

$$P(z) = \sum_{n=0}^{\infty} p_n z^n \qquad (7)$$

Multiplying each of the equations (1), (2), (3), and (5) by the appropriate power of z and taking $\sum_{n=0}^{\infty}$ for the resulting equations, it obtains:

$$z\sum_{n=0}^{\infty} p_n z^n = (p_0 + p_1)z \sum_{n=0}^{\infty} \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)z]^n}{n!}$$

$$+ p_2 z^2 \sum_{n=1}^{\infty} \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)z]^{n-1}}{(n-1)!}$$

$$+ p_3 z^3 \sum_{n=2}^{\infty} \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)z]^{n-2}}{(n-2)!} + \ldots \qquad (8)$$

From equations (7) and (8), and some algebra, to find:

$$P(z) = \frac{p_0(1-z)}{1 - ze^{\lambda(1+\eta)(1-z)}} \qquad (9)$$

Using the fact that $P(1) = 1$, along with LHopitals rule, it found:

$$p_0 = 1 - \lambda(1+\eta) \qquad (10)$$

From equation (7), (8) and $10)$, obtained as:

$$\sum_{n=0}^{\infty} p_n z^n = (1-z)[1-\lambda(1+\eta)] \sum_{i=0}^{\infty} e^{i\lambda(1+\eta)} e^{-i\lambda(1+\eta)z} z^i \qquad (11)$$

Substituting equations (7) and (10) into (11) and using $\sum_{i=0}^{\infty}\sum_{n=i}^{\infty} = \sum_{n=0}^{\infty}\sum_{i=0}^{n}$, we obtain as:

$$\sum_{n=0}^{\infty} p_n z^n = [1-\lambda(1+\eta)]\sum_{n=0}^{\infty}\left[\sum_{i=1}^{n} e^{i\lambda(1+\eta)}(-1)^{n-i}\frac{[i\lambda(1+\eta)]^{n-i}}{(n-i)!}\right.$$
$$\left.-\sum_{i=1}^{n-1} e^{i\lambda(1+\eta)}(-1)^{n-i-1}\frac{[i\lambda(1+\eta)]^{n-i-1}}{(n-i-1)!}\right]z^n \qquad (12)$$

Comparing the coefficients of z^n on both sides in equation (12), it finds:

$$p_n = [1-\lambda(1+\eta)]\left\{e^{n\lambda(1+\eta)} + \sum_{i=1}^{n-1} e^{i\lambda(1+\eta)}(-1)^{n-i}\right.$$
$$\left.\times\left(\frac{[i\lambda(1+\eta)]^{n-i}}{(n-i)!} + \frac{[i\lambda(1+\eta)]^{n-i-1}}{(n-i-1)!}\right)\right\},\ n \geq 2 \qquad (13)$$

From equation (1) and (10), obtained as:

$$p_1 = [1-\lambda(1+\eta)]\left(e^{\lambda(1+\eta)} - 1\right) \qquad (14)$$

4-Measures of effectiveness

To calculate the expected number of units in the system, using as:

$$L = E(n) = \sum_{n=0}^{\infty} n p_n \qquad (15)$$

Consider

$$A = \sum_{n=0}^{\infty} n^2 p_n \qquad (16)$$

Substituting equation (6) into (16), it finds:

$$A = B + C \qquad (17)$$

where

$$B = e^{-\lambda(1+\eta)} \sum_{n=0}^{\infty} n^2 \sum_{i=0}^{n} \frac{[\lambda(1+\eta)]^i}{i!} p_{n-i+1} \tag{18}$$

and

$$C = [1-\lambda(1+\eta)] e^{-\lambda(1+\eta)} \sum_{n=0}^{\infty} n^2 \frac{[\lambda(1+\eta)]^n}{n!} \tag{19}$$

Solving equation (18), obtained as:

$$C = \lambda(1+\eta)\left\{1-[\lambda(1+\eta)]^2\right\} \tag{20}$$

By solving equation (18), he gets:

$$B = \sum_{i=0}^{\infty} \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)]^i}{i!} \sum_{m=0}^{\infty} m^2 p_m$$

$$+2\sum_{i=0}^{\infty} \frac{ie^{-\lambda(1+\eta)}[\lambda(1+\eta)]^i}{i!} \sum_{m=0}^{\infty} mp_m$$

$$-2\sum_{i=0}^{\infty} \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)]^i}{i!} \sum_{m=0}^{\infty} mp_m$$

$$+\sum_{i=0}^{\infty} \frac{i^2 e^{-\lambda(1+\eta)}[\lambda(1+\eta)]^i}{i!} \sum_{m=1}^{\infty} p_m$$

$$-2\sum_{i=0}^{\infty} \frac{ie^{-\lambda(1+\eta)}[\lambda(1+\eta)]^i}{i!} \sum_{m=1}^{\infty} p_m$$

$$+\sum_{i=0}^{\infty} \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)]^i}{i!} \sum_{m=1}^{\infty} p_m \tag{21}$$

From equation (15), (16), (17), (19) and (21), he found:

$$L = \lambda(1+\eta) + \frac{[\lambda(1+\eta)]^2}{2[1-\lambda(1+\eta)]} \tag{22}$$

Also, calculate the expected number of units in the queue, using as:

$$L_q = \frac{[\lambda(1+\eta)]^2}{2[1-\lambda(1+\eta)]}, \quad (23)$$

So, calculate the expected waiting time in the system, using as:

$$W = 1 + \frac{\lambda(1+\eta)}{2[1-\lambda(1+\eta)]} \quad (24)$$

And, Calculate expected waiting time in the queue, using as:

$$W_q = \frac{\lambda(1+\eta)}{2[1-\lambda(1+\eta)]} \quad (25)$$

Remark. If the service times of the customers are exponential random variables with rate $\mu_n = \mu$ and $\eta = 0$,then the queue: M/M/1 without any concepts are all coincides with the results of Groos and Harris [3].

5- **Numerical illustration**

In this chapter, we randomly assign values for the parameters λ, and η to see the effect of their movement on the queue M/D/1. And that is as follows:

Assume that $\eta = 0.10$, then substituting these values into equations (10), (13-14), and (22-25) respectively was entered for the Mat CAD program, one obtain the results shown in Tables 1:

Table 1: The computed values of the M / D / 1 queue characteristics with $\eta = 0.10$ and changing the values of the parameter λ.

λ	P_0	P_1	P_2	P_3	P_4	L	L_q	W	W_q
0.05	0.945	0.053	0.00154	0.00003	0.00000	0.057	0.0016	1.029	0.029
0.10	0.890	0.103	0.00624	0.00027	0.00001	0.117	0.0068	1.062	0.062
0.15	0.835	0.150	0.01400	0.00097	0.00006	0.181	0.0160	1.099	0.099
0.20	0.780	0.192	0.02500	0.00249	0.00021	0.251	0.0310	1.141	0.141
0.25	0.725	0.229	0.04000	0.00520	0.00060	0.327	0.0520	1.190	0.19
0.30	0.670	0.262	0.05700	0.00954	0.00145	0.411	0.0810	1.246	0.246
0.35	0.615	0.289	0.07600	0.01600	0.00305	0.506	0.1210	1.313	0.313
0.40	0.560	0.310	0.09800	0.02500	0.00585	0.613	0.1730	1.393	0.393
0.45	0.505	0.323	0.12100	0.03700	0.01000	0.738	0.2430	1.490	0.49
0.50	0.450	0.330	0.14300	0.05100	0.01700	0.886	0.3360	1.611	0.611
0.55	0.395	0.328	0.16400	0.06800	0.02700	1.068	0.4630	1.766	0.766
0.60	0.340	0.318	0.18100	0.08700	0.04000	1.301	0.6410	1.971	0.971
0.65	0.285	0.298	0.19200	0.10600	0.05600	1.612	0.8970	2.254	1.254

For further clarification, the contents of Table 1 have beengraphically represented as follows:

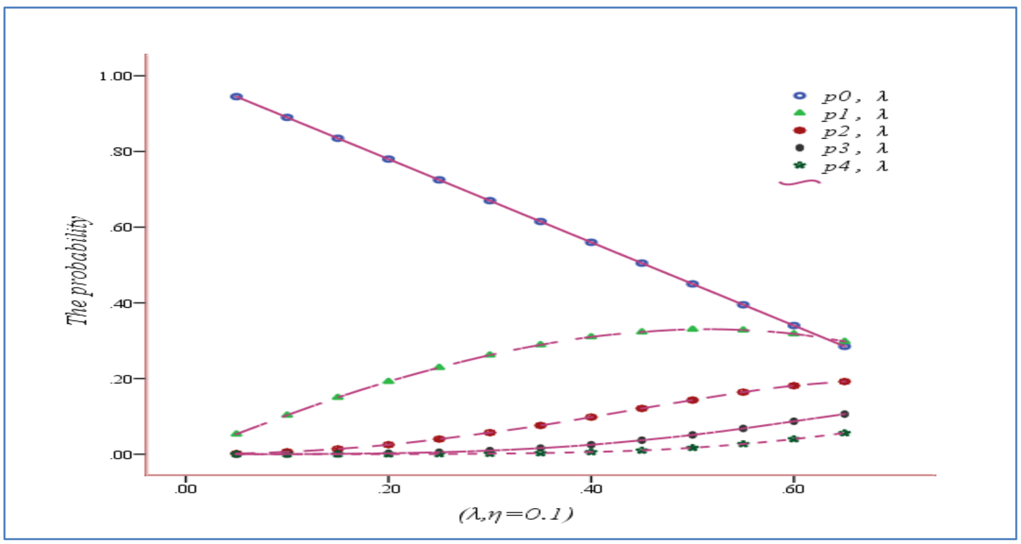

Figure 1: the relation $p_n == (0,1,2,3,4)$ & λ

From Table 1 and Figure 1, can notice that an increase in the access rate leads to a decrease in the probability of having no clients in the system, and an increase in the presence of *n* of clients in the system. That is, the results are consistent with the work of the model.

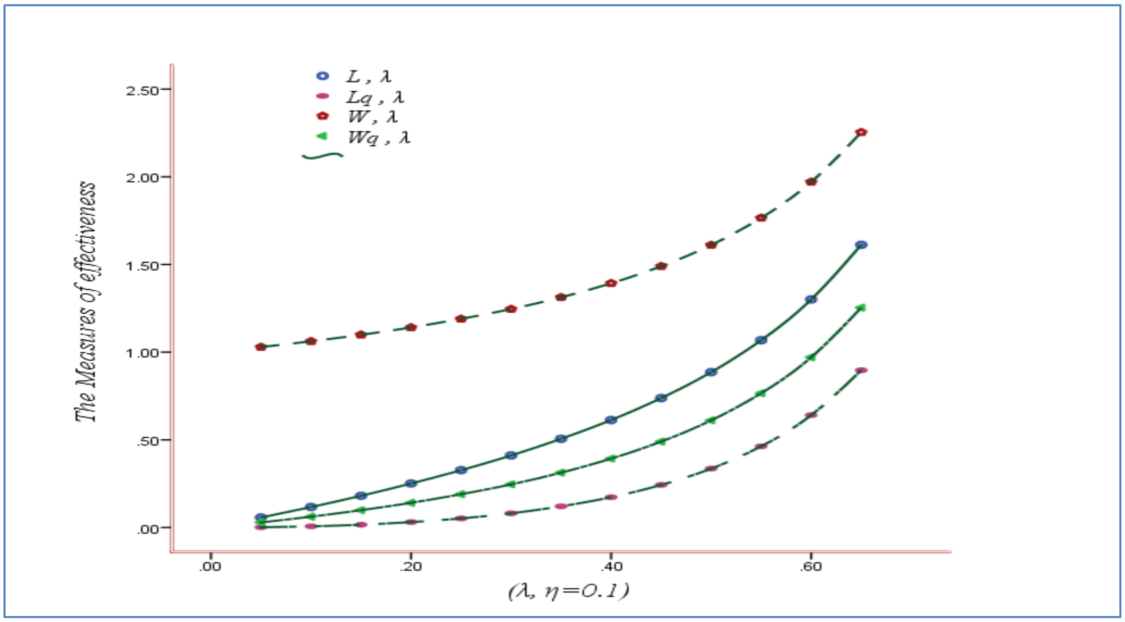

Figure 2: the relation L, L_q, W, W_q & λ

From Table 1, Figure 2, it is clear that the increase in average access rate contributes to the increase in the measures of effectiveness.

Also, assume that $\eta = 0.50$, one obtain the results shown in Table 2:

Table 2: The computed values of the M / D / 1 queue characteristics with $\eta = 0.50$ and changing the values of the parameter λ.

λ	P_0	P_1	P_2	P_3	P_4	L	L_q	W	W_q
0.05	0.925	0.072	0.00288	0.00008	0.00000	0.078	0.00304	1.041	0.041
0.10	0.850	0.138	0.01200	0.00072	0.00004	0.163	0.01300	1.088	0.088
0.15	0.775	0.196	0.02700	0.00268	0.00023	0.258	0.03300	1.145	0.145
0.20	0.700	0.245	0.04700	0.00694	0.00091	0.364	0.06400	1.214	0.214
0.25	0.625	0.284	0.07300	0.01500	0.00268	0.487	0.11300	1.300	0.300
0.30	0.550	0.313	0.10200	0.02700	0.00653	0.634	0.18400	1.409	0.409
0.35	0.475	0.328	0.13300	0.04400	0.01400	0.815	0.29000	1.553	0.553
0.40	0.400	0.329	0.16200	0.06700	0.02600	1.050	0.45000	1.750	0.750
0.45	0.325	0.313	0.18400	0.09200	0.04400	1.376	0.70100	2.038	1.038
0.50	0.250	0.279	0.19400	0.11700	0.06800	1.875	1.12500	2.500	1.500
0.55	0.175	0.224	0.18200	0.13000	0.09000	2.770	1.94500	3.357	2.357
0.60	0.100	0.146	0.13800	0.11500	0.09400	4.950	4.05000	5.500	4.500
0.65	0.025	0.041	0.04500	0.04400	0.04200	19.988	19.013	20.500	19.50

For further clarification, the contents of Table 2 have been graphically represented as follows:

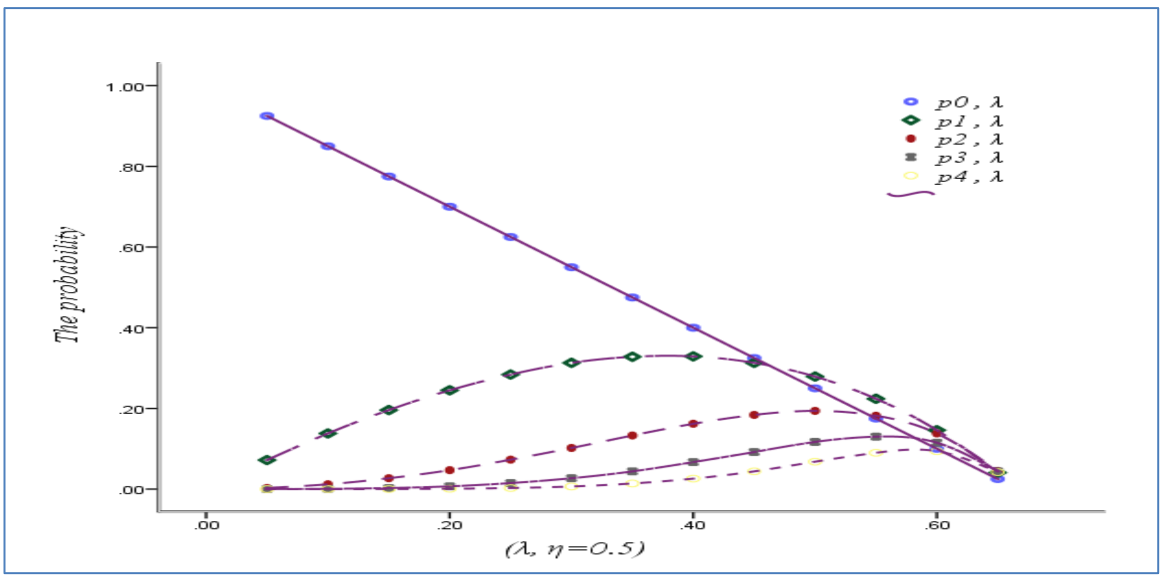

Figure 3: the relation $p_n == (0,1,2,3,4)$ & λ

From Table 2 and Figure 3, can notice that an increase in the access rate leads to a decrease in the probability of having no clients in the system, and an increase in the presence of n clients in the system. Also, we note that an increase in the encouragement of arrivals leads to an increase in the probabilities.

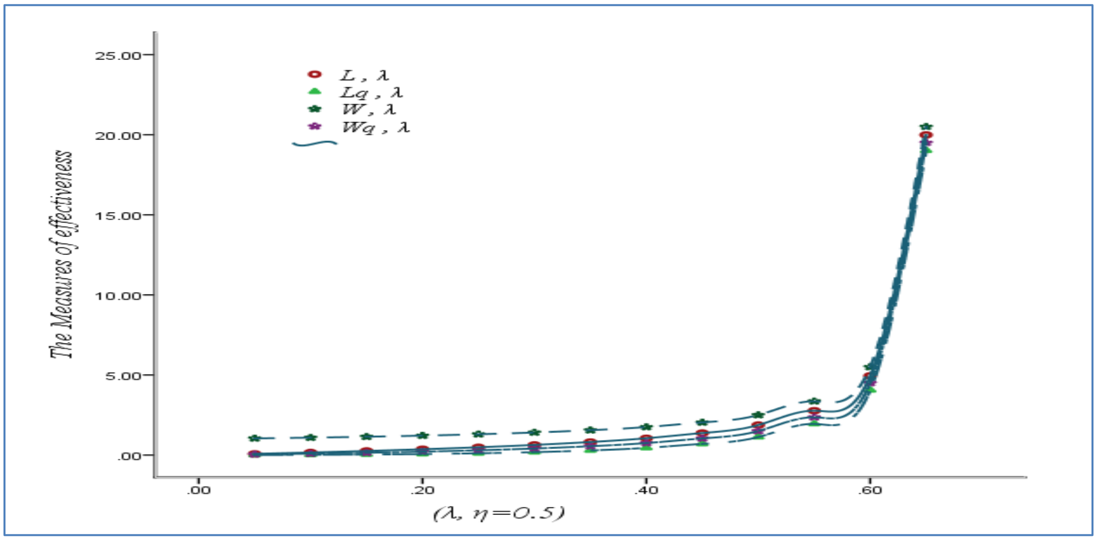

Figure 4: the relation L, L_q, W, W_q & λ

From Table 2, Figure 4, it is clear that the increase in average access rate contributes to the increase in the measures of effectiveness. Also, we note that the increased encouragement of arrivals leads to a significant increase in effectiveness measures.

6-Conclusion and recommendation

This paper extracted the steady-state solution of the M / D / 1 queue with encouraging arrivals. The probabilities of having n customers in the system, some performance measures. The arrival rate and encouraging arrivals have a positive effect. That is, the more the company manages to increase the percentage of the increase in the number of customers the more customers will trust it, and this happens through deliberate discounts on goods. For future work, the queue M/D/1/N can be studied with the addition of encouraging arrivals.

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