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EFFECTIVE MEDIUM INTRASUBBAND SURFACE PLASMON-POLARITONS ON SEMI-INFINITE GaAs/AlAs SUPERLATTICES

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The effective medium theory of the long wavelength surface electromagnetic excitations of a multi-layered medium is employed to describe the surface plasmon-polaritons of a semi-infinite array of strictly two-dimensional charge sheets. A virtual p-polarised surface plasmon polariton is predicted and its associated attenuated total reflection (ATR) reflectivity calculated for typical parameters. The existence criteria for this surface polariton are less restrictive than for the Giuliani and Quinn surface mode. A radiative Brewster mode is also found and this mode may be observed via oblique incidence reflectivity.

Keywords: A. dielectric response, B. quantum wells, C. inelastic light scattering

1. INTRODUCTION.

The electromagnetic modes of layered media have been of interest both theoretically and experimentally [1] over the past few years. In the long wavelength regime where the wavelength of the electromagnetic excitation, λ, is much greater than the superlattice period, D, the exact dispersion relations can be expanded in powers of D/λ leading to an effective medium dielectric tensor [2,3]

$$\tilde{\varepsilon} = \begin{pmatrix} \varepsilon_{\parallel} & 0 & 0 \\ 0 & \varepsilon_{\parallel} & 0 \\ 0 & 0 & \varepsilon_{\perp} \end{pmatrix} \quad (1)$$

In (1) $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ describe the response of the medium to electromagnetic fields parallel and perpendicular to the interfaces respectively. The above tensor illustrates that in this long wavelength regime the superlattice has the optical characteristics of a uniaxial medium and one can then employ the well established theory for the optics of materials with uniaxial symmetry. Substantial theoretical and experimental investigations on the polariton modes of layered media in this effective medium limit has been carried out by Tilley and co-workers [1,4,5]. For brevity some theoretical results which are presented in these articles are employed here without derivation.

The elements of the effective medium tensor are expressible in terms of the bulk dielectric functions of the constitutive materials. To be more precise, the superlattice unit cell is composed of two materials of width a and b, and has a total width $D = a + b$. The constituent layers have dielectric functions ε_a and ε_b which are in general frequency dependent. The elements of the effective medium dielectric tensor (1) are [1-3]

$$\varepsilon_{\parallel} = \varepsilon_a f_a + \varepsilon_b f_b \quad (2)$$

$$\varepsilon_{\perp} = \frac{\varepsilon_a \varepsilon_b}{f_b \varepsilon_a + f_a \varepsilon_b} \quad (3)$$

with $f_a = a/D$ and $f_b = b/D$ the volume fractions occupied by materials a and b.

The above outline of the theory is necessarily brief and the details may be found elsewhere [1-3]. It is noted here that for various choices of ε_a and ε_b the surface phonon-polaritons and the surface plasmon-polaritons have been calculated using (1)-(3) and their resultant spectral properties successfully compared with experiment [1].

The purpose of this Communication is to demonstrate that the above effective medium theory applies to the description of surface plasmon-polaritons of a semi-infinite superlattice of strictly two dimensional (2D) charge sheets. The frequencies of the surface modes lie to the right of the light-line and as such are not observable by direct optical reflection measurements. Attenuated total reflection (ATR), on the other hand, is an ideal spectroscopic technique to investigate such surface modes [1,4-6], and their ATR spectrum will also be described here.

The plasmon oscillation of an infinite array of 2D charge sheets has the well known linear dependence of frequency on in-plane wavevector for fixed non-zero wavevector normal to the surface [7,8]. This theoretical prediction was substantially confirmed by the Raman measurements of Olego et al [9]. Giuliani and Quinn [10] first predicted the surface plasmon of a semi-infinite superlattice of 2D charge sheets. These surface modes exist in regions where retardation is unimportant, and are free of Landau damping. It is fair to say that up until now they have not been unambiguously observed, although substantial experimental effort in this field has been carried out [11,12]. The reason lies in the restrictive existence criteria for these modes. In particular, the surface mode does not exist if the capping layer thickness is greater than half the superlattice thickness, $D/2$ [13,14]. The tendency for the quantum well nearest the surface to deplete is, therefore, a substantial practical drawback as was highlighted by Pinczuk and co-workers [11].

In this work we describe the retarded surface plasmon-polaritons of a semi-infinte 2D sheets system within the effective medium model.

2. DISPERSION RELATION.

The charge sheets model can formally be obtained within the effective medium model as follows. Layer a is assumed to contain n electrons per unit volume, whilst layer b is undoped. The bulk long wavelength dielectric functions which govern the responses of these two layers are therefore

$$\varepsilon_a = \varepsilon\left(1 - \frac{\omega_p^2}{\omega^2}\right) \qquad \varepsilon_b = \varepsilon \tag{4}$$

In (4) ε is a background dielectric constant ($\varepsilon = 13$ appropriate for GaAs) and $\omega_p = \sqrt{ne^2/m^*\varepsilon_o\varepsilon}$ is the 'bulk' plasma frequency of the doped layer (all other symbols have their customary meaning). The background dielectric constants are assumed to be the same for the two layers, which is not too bad an approximation for the GaAs/AlAs system considered here. The optic phonon contributions to the dielectric functions are ignored and this is justified later.

The use of a bulk plasmon response (4) for the doped layer a has been applied within the effective medium model in previous investigations [1]. This approximation obviously breaks down when quantum confinement of the carriers in the growth direction is important (i.e. when the width of the well is of the order of the electron wavelength typically ≤ 300Å). In such situations, the dielectric function should be calculated microscopically, for example within the random phase approximation (RPA), to include subband effects. This is beyond the scope of the present work. Alternatively, when the width of the doped layer is small such that only the first subband is occupied, and the interest is in intrasubband plasmons, then the assumption of a *strictly* 2D electron sheet gives the same results (in the long wavelength limit) for the plasmon frequency whether calculated within RPA [10] or classically via Maxwell's equations [e.g.7,15,16]. This strictly 2D limit is now taken within the effective medium model.

Formally the limit $a \to 0$ is carried out in (2) and (3) and this leads to

$$\varepsilon_{\|} = \varepsilon\left(1 - \frac{\Omega_p^2}{\omega^2}\right) \qquad \varepsilon_{\perp} = \varepsilon \tag{5}$$

by noting that

$$f_a\omega_p^2 \to \frac{n_s e^2}{m^* D\varepsilon_0\varepsilon} = \Omega_p^2 \tag{6}$$

with n_s the *sheet* carrier concentration. In (5) and (6) $\hbar\Omega_p$ is the typical 2D plasma energy, and for the parameters appropriate for sample 1 of Olego et al ($n_s = 7.3\times10^{15}\,\mathrm{m}^{-2}$ and $D = 890$Å) is 11.4meV. This justifies the neglect of the optic phonon contributions to the dielectric tensor since this value for Ω_p lies well below the reststrahl bands of both GaAs and AlAs. The elements of the effective medium tensor (5) can, alternatively be obtained from the dispersion relation for the bulk plasmon polaritons of an infinite array of 2D charge sheets (see Appendix).

An important observation regarding the effective medium model is noted here. In the long wavelength regime in which it applies, layer a or layer b can be the terminating layer of the superlattice, it makes no difference. It does, of course make a difference to the electrostatic surface modes since these modes have in-plane wavevectors of order $1/D$, hence the penetration length is of order the superlattice period. Within the context of the charge sheets model, the uppermost charge sheet can either be at the surface as is the case for the surface plasmon predicted by Giuliani and Quinn [10], or be a distance D below the surface. The latter corresponds to a capping layer of width D which is more convenient from the experimental point of view as a consequence of the depletion problem outlined in the previous section. The superlattice is therefore assumed to terminate at $z = 0$ and consist of charge sheets at $z = -mD$ where m is a positive non-zero integer. For such a system the Giuliani and Quinn surface mode does not exist [13,14] when the medium $z > 0$ is vacuum as assumed here.

Polaritons localised at the surface of the superlattice, and which are p-polarised, obey the following dispersion relation [e.g.5]

$$q^2 = q_0^2\left[\frac{(\varepsilon_{\|} - \varepsilon_m)\varepsilon_m\varepsilon_{\perp}}{\varepsilon_{\|}\varepsilon_{\perp} - \varepsilon_m^2}\right] \tag{7}$$

with q the propagation wavevector along the interface, $q_0 = \omega/c$ (the wavevector in vacuum), and ε_m the dielectric constant of the medium occupying $z > 0$. The localisation conditions imply that the surface mode exists if (since $\varepsilon_{\perp} > 0$)

$$q > q_0 \text{ and } \varepsilon_{\|} < 0 \tag{8}$$

From the general properties of the system [5], the surface mode is virtual, in other words it does not persist to the unretarded (electrostatic) limit, and has maximum in-plane wavevector $q = \sqrt{\varepsilon_{\perp}}\,q_0$.

Although our interest is primarily with the localised surface plasmon polariton, the dispersion relation (7) also describes radiative modes of the system known as Brewster modes [5,17]. Brewster modes lie to the left of the light-line $q < q_0$, and hence, ordinary reflectivity experiments can in principle detect these modes. By a straightforward manipulation of (7), it can be verified that the Brewster mode

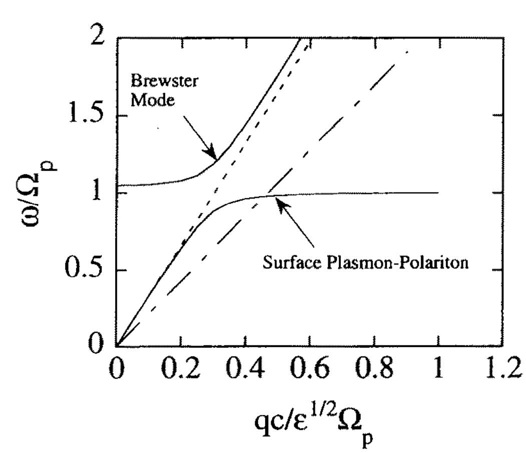

Figure 1. The dispersion relation for the surface plasmon-polariton and the radiative Brewster mode. The dotted curve is the vacuum light line $\omega = cq$, and the dot-dashed curve the ATR scan line $\omega = cq\varepsilon_p^{-1/2}\operatorname{cosec}(\theta)$, with ε_p the dielectric constant of the prism and θ the angle of incidence.

has a frequency at zero in-plane wavevector of $\Omega_p\sqrt{\varepsilon/(\varepsilon-\varepsilon_m)}$. For large in-plane wavevectors, the Brewster mode frequency is asymptotic to the line

$$\omega=\sqrt{\frac{\varepsilon_m+\varepsilon}{\varepsilon_m\varepsilon}}cq \tag{9}$$

Figure (1) illustrates the dispersion curves for both the non-radiative surface mode and the radiative Brewster mode for a semi-infinite superlattice of strictly 2D charge sheets terminating at vacuum ($\varepsilon_m=1$). Also shown on this figure is the vacuum light line and the so-called ATR scan line which is required in the following section.

3. ATR AND REFLECTIVITY SPECTRA

The ATR and oblique incidence reflectivity spectra associated with the modes described in section 2 are now determined numerically by utilising previous theoretical results. For brevity, these are not re-derived.

In order to observe the surface plasmon polariton, an ATR experiment is required since the modes frequency lies to the right of the vacuum light line. Briefly, in such an experiment [1,6], incident radiation is reflected from the base of a prism at such an angle as to achieve total internal reflection. When the prism is brought near to the surface of the superlattice the evanescent wave at the base of the prism can couple to the superlattice surface mode if the energy and in-plane momentum are matched. It is calculated, although the details are not given, that for a Si prism and incident radiation at an angle of 30° to the normal, corresponding to the ATR scan line in figure (1), the optimum distance from the base of the prism to the surface of the superlattice should be $10\mu m$. The expression for the ATR reflectivity is standard [e.g.4] although it is noted that damping has to be incorporated. This is achieved phenomelogically by the replacement $\omega^2\to\omega(\omega+i\gamma)$ in (5) where γ is a damping parameter. At low temperatures and for high mobility samples, the contributions to γ are due predominantly to the scattering of the carriers by remote impurities and interface roughness. For sample 1 of Olego et al it is estimated [18] that $\hbar\gamma\approx 0.3$ meV(corresponding to 2.4 cm^{-1}). The ATR reflectivity is illustrated in figure (2) for this value and also for a higher value of 1.25meV (10 cm^{-1}). For both a pronounced dip in the ATR reflectivity is predicted at the surface plasmon frequency.

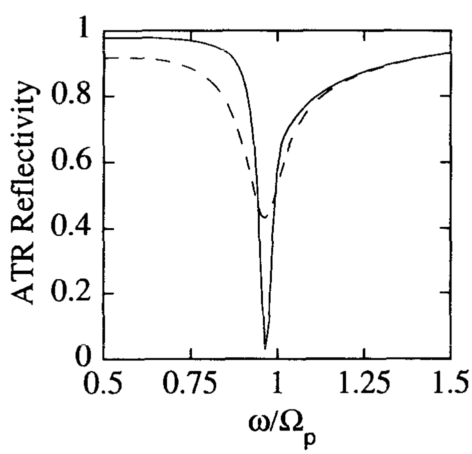

Figure 2. ATR reflectivity as a function of incident radiation frequency with $\theta=30^{\circ}$. The full curve corresponds to a damping parameter of 0.3meV whilst the dashed curve corresponds to a damping parameter of 1.25meV.

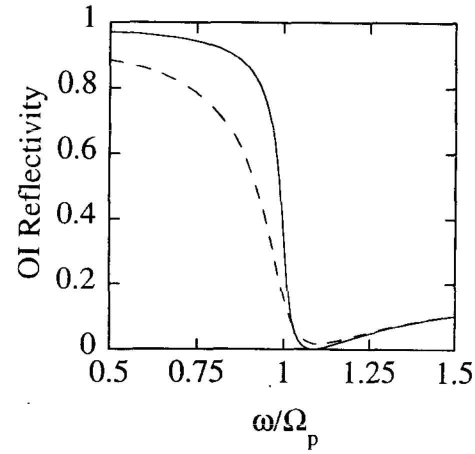

Figure 3. Oblique incidence reflectivity as a function of incident radiation frequency. The damping parameters for the two curves are as in figure 2 and the angle of incidence is 30°.

The Brewster mode lies to the left of the light line and can therefore be detected via direct oblique incidence reflectivity. By employing equation (13) of reference [5] the reflectivity at oblique incidence can be calculated as a function of frequency for a given damping. Figure (3) depicts the oblique incidence reflectivity as a function of frequency for the two damping parameters employed in figure (2). A pronounced decrease in the reflectivity at the Brewster mode frequency is clearly observed for both damping parameters.

4. CONCLUSIONS

In this Communication it is demonstrated that the effective medium description of a layered system can be applied to describe the intrasubband surface plasmon-polaritons of a semi-infinite array of strictly 2D charge sheets. A virtual surface plasmon-polariton is predicted and its associated ATR spectrum calculated. Moreover, this mode exists within the model if the capping layer thickness is equal to the superlattice period D. This is less restrictive than the analogous criterion for the Giuliani and Quinn surface plasmon, and holds out the prospect of the observation of this virtual surface mode via ATR spectroscopy in suitably constructed samples.

A radiative Brewster mode is also predicted and its associated oblique incidence reflectivity calculated for typical plasmon damping. This mode should also be observable and it is hoped that this work will stimulate experimental efforts in this direction.

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APPENDIX

The dispersion relation for bulk p-polarised plasmon polaritons of an infinite array of 2D charge sheets is given by [15,16]

$$\frac{\omega^2}{\Omega_p^2} = \frac{(\tilde{q}D)\sinh(\tilde{q}D)}{2[\cosh(\tilde{q}D) - \cos(q_z D)]} \quad \text{(A1)}$$

where $\tilde{q} = \sqrt{q^2 - \varepsilon q_0^2}$ and q_z is the Bloch wavevector normal to the interfaces $(0 \le q_z D \le \pi)$. In the long wavelength limit $\tilde{q}D << 1$ and $q_z D << 1$ (A1) reduces to

$$q^2 \varepsilon_\perp^{-1} + q_z^2 \varepsilon_\parallel^{-1} = q_0^2 \quad \text{(A2)}$$

with $\varepsilon_\parallel$ and $\varepsilon_\perp$ given by (5). It is seen from (A2) that in this limit the infinite array of 2D sheets behaves as a conventional uniaxial medium.

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