

# REDUCTION OF THE RESPONSE OF FLEXIBLE STRUCTURES TO THE UNDER-PRESSURE OF BLAST LOADING

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## المخلص

إن تقييم استجابة المنشآت المعرضة لأحمال الانفجارات أصبح نقطة بحث للمهندسين المدنيين منذ عقود. الهدف من هذه الورقة البحثية هو الأخذ في عين الاعتبار منطقة الضغط السالب عند تمثيل منحنى الضغط مع الزمن لحمل الانفجار ومعرفة تأثير الضغط السالب على استجابة المنشآت نتيجة تعرضها لأحمال الانفجار. تتناول هذه الورقة المنشآت المرنة التي يكون زمنها الدوري الطبيعي عالي. لغرض حساب الإزاحة للمنشأ يتم تحويله بالكامل إلى نظام أحادي درجة الحرية (SDOF)، حيث تعتبر هذه الطريقة التطبيقية هي الأكثر شيوعاً في حساب الإزاحة الديناميكية، وتم إجراء الحسابات اللازمة لمعرفة استجابة المنشأ باستخدام برنامج تم تصميمه خصيصاً. ومن النتائج التي تم الحصول عليها عند أخذ المنطقة السالبة في منحنى حمل الانفجار في الحسبان ملاحظة تقليص في إزاحة المنشأ مقارنة بما لو أن منحنى حمل الانفجار تم تمثيله بدون منطقة الضغط السالب. كما لوحظ أنه مع زيادة نسبة المقاومة القصوى للمنشأ الي مقدار قوة الانفجار فإن معامل التخفيض في الإزاحة يزيد أيضاً. من النتائج المتحصل عليها في هذه الورقة يمكن إخراج جداول بيانية معدلة يمكن من خلالها معرفة الإزاحة القصوى المتوقعة لمنشأ ذو أبعاد وخواص معينة وعند أي سيناريو تفجير.

## ABSTRACT

The assessment of structural response due to blast loading has become a point of research and interest for blast civil engineers for decades. The aim of this paper is to include the negative phase in the modelling of the blast load and to investigate the effects of under-pressure (Negative phase) resulting from impact loading of an explosion on structural response. This paper have targeted a typical structure with high natural period in other words flexible structure. Exponential approach has been used for modelling the pressure-time history of the blast load, where the pressure decays non-linearly. In order to determine the structural deformation, a numerical model of single degree of freedom system (SDOF) was used. resulting of including the negative phase showed a noticeable reduction in the peak deflection in the case of the flexible structure response. In addition, reduction factor of structural deformation have increased as the ratio of the ultimate structural resistance to the blast force ( $R_u/P$ ) increased. Therefore, the negative phase should be included in the blast load modelling even if it is considered conservative. New modified charts have been extracted, which engineers may use to predict the reduction value in structural deformation, if under-pressure was added. Reduction charts have been classified based on the structure member property and geometry at any explosion scenario.

**KEYWORDS:** Blast Load; Negative Phase; Under-Pressure; SDOF; Reduction Factor; Structural Response; Peak Deflection.

## **INTRODUCTION**

The assessment of blast loads has become a point of interest and research for engineers after World War I. Even though, in a terrorist bomb attack, it is unfeasible to predict the magnitude of blast load which depends on several factors; magnitude of explosion charge, location of detonation epicentre relative to the structure, geometrical configuration of structure, direction and height of structure with respect to the explosion centre and ground surface. Blast engineers are facing a challenge of modelling the impact loading more accurate, due to unpredictable interaction among blast waves during the explosion, which may result in more than a single shock wave that may hit the structure. This paper will concentrate on including the negative part of the blast loading, which is known as under pressure in the pressure-time history curve in case where the structure would receive a single blast wave. Furthermore, focusing on flexible structures, in which all structural members have high natural period of time (T), which is the time that is needed to complete one cycle of vibration. A practical example of such structures is glazed panel with thin thickness and large dimension that are being used nowadays in the modern architecture.

Most of former studies that have been conducted to study the structure response resulted from the blast impact; positive phase only was being included in pressure-time history of blast loading. The negative phase was neglected, and has been considered as a conservative, if it was included. Structure deformation was represented by a single degree of freedom (SDOF) system. SDOF model is becoming the most desirable method to use by analyst in term of finding the dynamic response of a structure [1].

An explanation of the use of the equivalent SDOF method and the assumptions that should be made to determine structural response are given below along with the evaluation of the structural response due to the blast load, where the curve of pressure-time history is modelled decays non linearly. Comparison was made between maximum deformation that was resulted from blast load where negative phase is included, and blast load without negative phase in the curve of pressure-time history.

## **METHODOLOGY**

Structure deformation due to blast loading could be conveniently represented by single degree of freedom (SDOF) system. In this simplified approach model, a single physical degree of freedom is used in order to determine the deformation of a structural element. The SDOF model can be considered as an idealized supposition, which could be classified in three parts: material assumption, loading assumption and geometry assumption. Due to the simplicity of SDOF model, it became the most favourable method to use in finding the dynamic response of a structure [1].

### **Equivalent (SDOF) System**

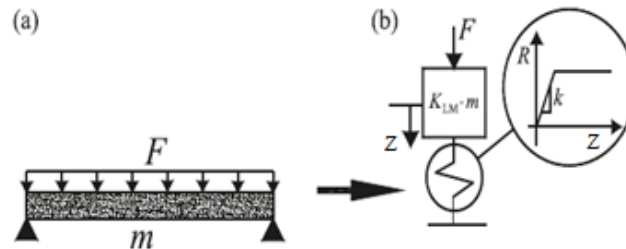
The simplest way of modelling the structure is to equivalent single degree of freedom system SDOF. In this system where mass of structure translated to lumped mass as well as load and stiffness as it is shown in Figure (1). The result from using SDOF system is one dimensional deformation. In addition, equivalent dynamic factors will be used in order to obtain the equivalent system. These factors relate to deformation, Kinetic

energy and work done by external force and are obtained by the following transformation formulas:

$$M_E = K_M M \quad , \quad F_E = K_L P \quad , \quad R_E = K_R R$$

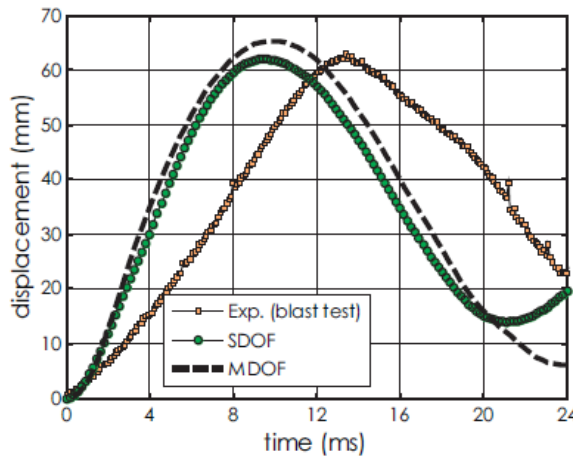
Where:

$M_E$ ,  $F_E$  and  $R_E$  are equivalent mass, load and resistance respectively.



**Figure 1: Single Degree of Freedom system: a) Structural member. b) Equivalent SDOF system [2]**

In Figure (2), the experimental time-history of the beam in shot 5 is compared with the corresponding SDOF and MDOF models predictions based on the above assumption. The peak displacement predicted by the preceding models are 62 and 65.2 mm, respectively, differing by 1.8% and 3.1% from the corresponding experimental value. From these results, it is clear that both models predict the experimental displacement of each beam up to the peak displacement reasonably well and the difference between the predictions of the two models is relatively small.



**Figure 2: Displacement time history of blast loading Shot 5: W=250 kg, S=9.5 m, Z=1.51 m/kg<sup>1/3</sup>. [3]**

The resistance function of the structure is modelled as an ideal elastic-plastic as illustrated in Figure (3) and represented in equation (1) [3].

$$R(K, R_{max}, x) = \left\{ \begin{array}{ll} Kx = \frac{R_{max}}{x_e} \cdot x & 0 < x \leq x_e \\ R_{max} & x > x_e \end{array} \right\} \dots \dots \dots (1)$$

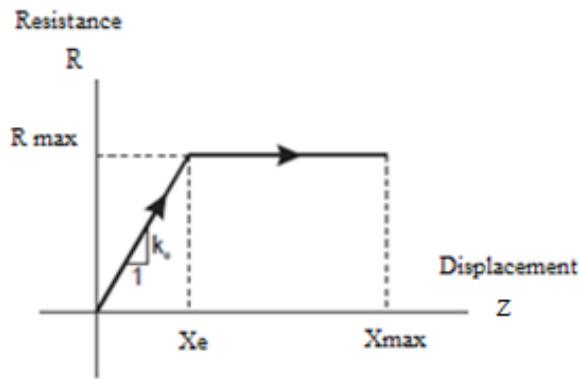


Figure 3: Elastic-plastic response spectra. [1]

### Methods of Modelling the Positive and Negative Phases for Blast Loads

Pressure-time history of blast load for many years has been simplified by using linear decays modelled by a triangular in order to determine the structural response easily through explicit solution. However, due to the revolution in computational analysis devices such as MATLAB programme and other computer soft-wares, it can be easily now to solve and analyse problems of non-linear trends of blast loading or structural behaviour.

### Non-Linear approximation

The positive phase shape was modelled by using modified the Friedlander equation (2) [4], in which the load decays exponentially as shown in Figure (4). The cubic expression of the negative phase was given by Granström in the 1960s, as shown in equation (3) [5].

$$P_r(t) = \left\{ +P_{r,max} \left[ 1 - \frac{(t)}{t_d} \right] \times e^{-c \frac{(t)}{t_d}} \right\} \quad (2)$$

$$P_r(t) = \left\{ -P_{r,min} \left[ \frac{6.75 (t-t_d)}{t_d^-} \right] \left[ 1 - \frac{(t-t_d)}{t_d^-} \right]^2, \quad t_d < t \leq t_d + t_d^- \right\} \quad (3)$$

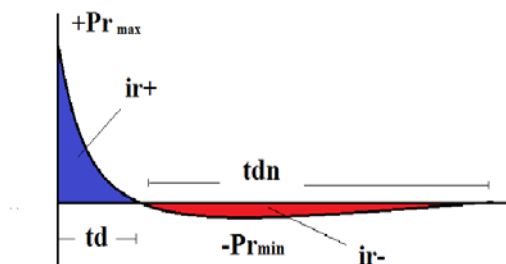
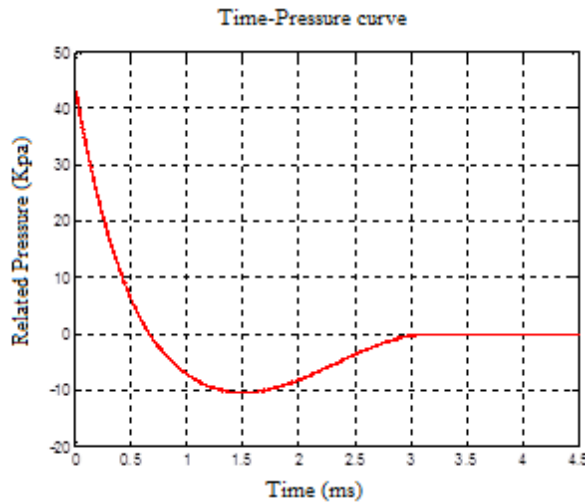


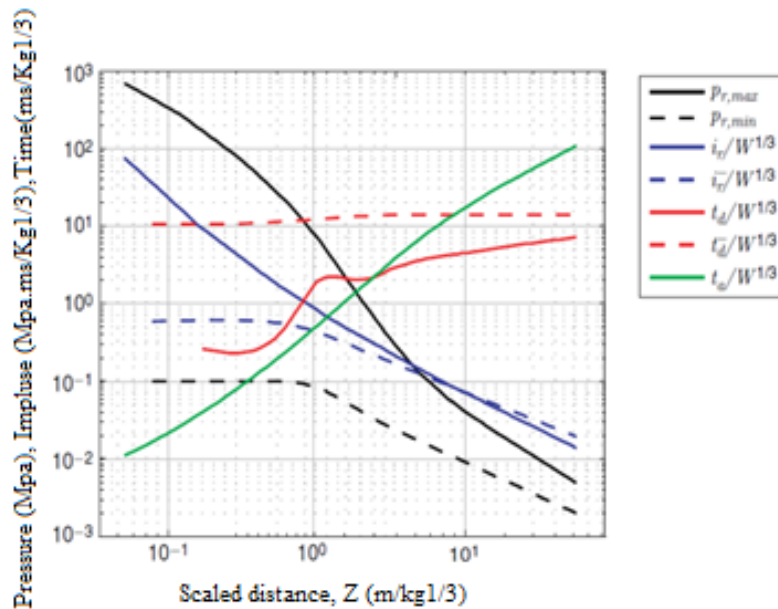
Figure 4: Exponential approximation of blast load.

Measuring or predicting the second shock wave is complex due to different factors of blast wave interaction time with obstacles. Figure (5) shows the shape of blast load generated from Math-lab programme that will be used for analysis, where the blast load has only a single shock wave.



**Figure 5: non-linear decay of blast loading with a (0.3kg) semi-spherical charge.**

The blast load parameter values such as magnitude of blast pressure, positive and negative impulse, time duration for both phases could be obtained directly from the chart below in Figure (6).



**Figure 6: Positive and negative phase reflected blast wave parameters for hemi spherical charges of TNT on the surface [4].**

### Implicit Average Acceleration Method

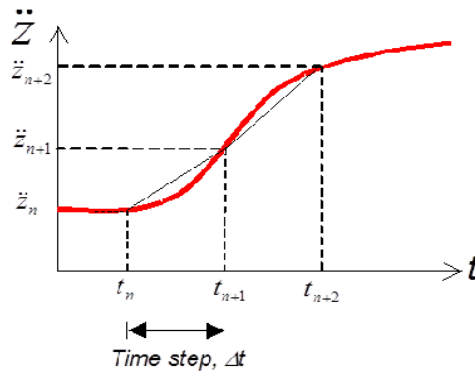
This method is applied in case of having non-linearity in blast load-time history or the resistance-deflection function. This method is an appropriate one to be used instead of solving equation of motion in traditional way by integration. This method, called Average Acceleration Method, assumes constant acceleration at small time increments ( $\Delta t$ ). In order to determinate the displacement, Simpson's method was used. This method known also as acceleration method, which can basically find the acceleration ( $\ddot{Z}$ ) from

equation (4). Velocity and displacement for each time can be calculated from equation (5) and (6) respectively.

$$\ddot{Z} = \frac{P_e(t)}{m_e} - \frac{K_e Z}{m_e} \quad (4)$$

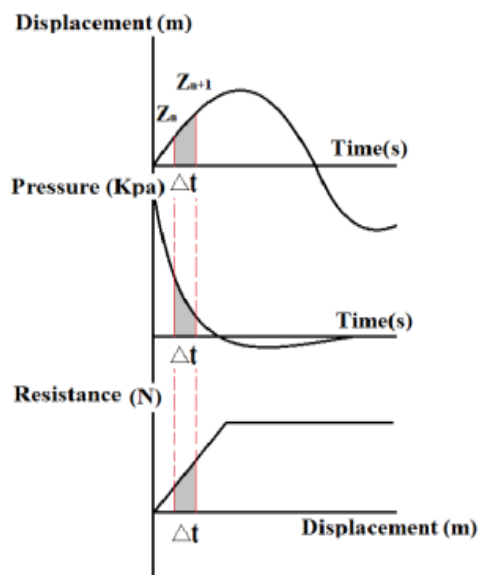
$$\dot{Z}_{n+1} = \dot{Z}_n + \int_{t_n}^{t_{n+1}} \ddot{Z} dt = \dot{Z}_n + \frac{(\ddot{Z}_n + \ddot{Z}_{n+1})\Delta t}{2} \quad (5)$$

$$Z_{n+1} = \frac{\left( Z_n + \dot{Z}_n \Delta t + \frac{\Delta t^2}{3} \ddot{Z}_n + \frac{P_e(t+1)\Delta t^2}{4m_e} \right)}{\left( 1 + \frac{K_e \Delta t^2}{4m_e} \right)} \quad (6)$$



**Figure 7: Average acceleration method.**

This simple procedure for dynamic response calculation is valid in case of pressure-time history for blast load is non-linear as well as resistance function of structure material. At any time scale ( $\Delta t$ ), the pressure and resistance has different values as it can be seen clearly in Figure (8). Using a numerical SDOF system, the structural response could be solved instantaneously at ( $\Delta t$ ) for any function. The optimization of the accuracy of this numerical method depends on the discretizing number.



**Figure 8: Time discretization for non-linear pressure and resistance functions.**

## COMPUTATIONAL RESULTS AND ANALYSIS

The negative phase has the capability to reduce structural response, and minimize the effective-cost of the structure that withstands the blast loading especially for flexible structures. The aim of this paper is to define the maximum deflection of the structural response, and to analyse the findings and compare them with structural response for cases of including and not including the negative phase. Also implementing approximation of non-linear blast pressure curve. It is also aimed to find the intensity of blast load pressure that can lead to structural collapse at first cycle of oscillation. Based on this concept, the first peak response of structure is the main target of this research.

### First Peak Response of Structure (Inward Response)

The most important point for blast engineers is to find the first maximum deflection of structural response. It is common to neglect the damping effect of structure element for blast load resisting due to high probability of structural failure member before even reach its first few cycles of displacement

A theoretical example of simply supported element with dimensions of (0.5 m, 0.02 m, and 1.0 m) for width, thickness and the length or height of the structural member respectively was tested under impact loading. The structural element had experienced a pressure wave from detonating a spherical charge (1 and 10) kg of TNT. Material properties of structural member are assumed to be for the Young's models ( $E=200$  MPa) and density ( $\rho = 7500$  kg/m<sup>3</sup>). Using a numerical SDOF the response of the structure is firstly investigated by modelling the only positive phase of blast pressure, and then negative phase is added to blast pressure-time history. Blast wave have assumed to interact fully with structural element surface at the same time to guarantee the pressure is equal at each point on structure surface. Figure (9) shows the interaction of blast wave with structure.

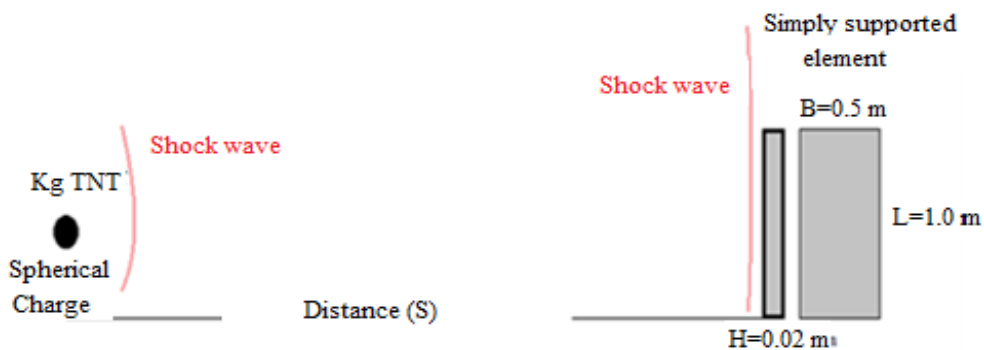


Figure 9: Blast wave interaction with the element.

Simplified positive phase loading and ignoring the negative phase loading was a practical consideration introduced in some military manuals such as UFC 3-340-02 [4]. Blast civil engineers used the chart in Figure (10) for the past 50 years to find and predict the maximum deformation of a structural member that is subjected to blast loading. The member that was used for the chart creation was a simply supported structural element. However, in this chart the blast load was represented by a linear triangular with only positive phase included. In addition, there was no accurate details for the structure geometry and property.

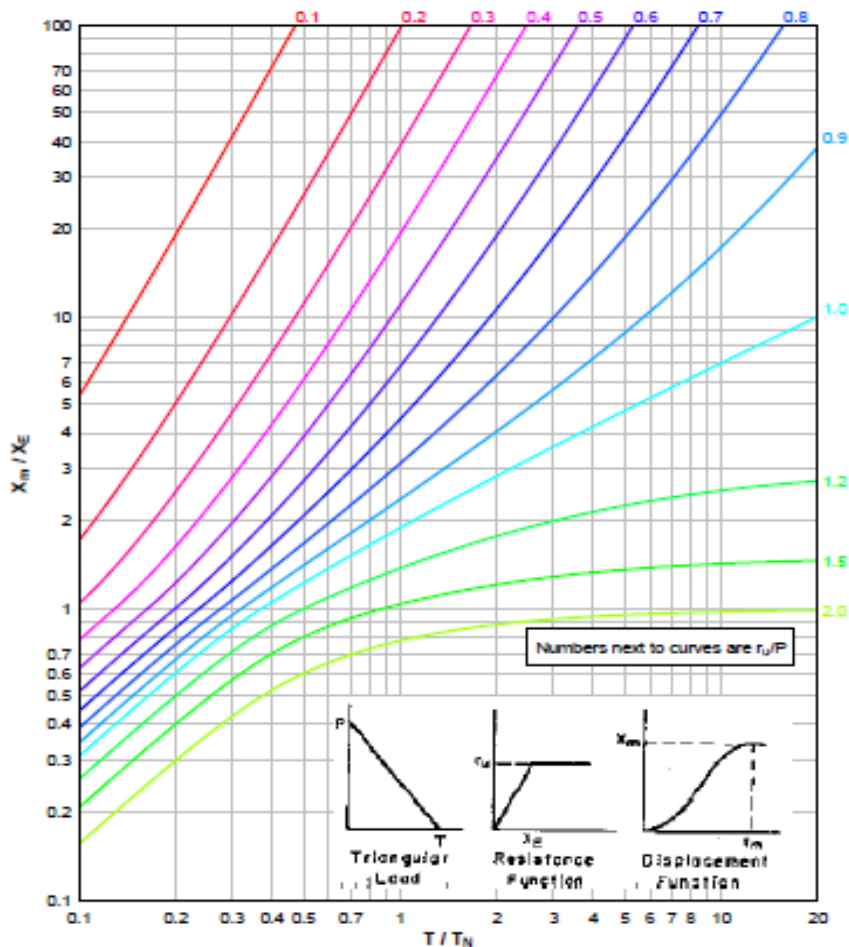


Figure 10: Typical chart to calculate maximum response  $X_m$  [4]

Hundreds of analyses have been made by using MATLAB code, in order to evaluate the structural response, because of blast loading with different possibilities and scenarios for both structure and explosion loads. Analyses have done for both cases where negative phase included and not included. Following steps will show the effect on structural response when the property and blast load scenarios were changed. These changes will give a verity of modification charts that could be used to find the reduction in the maximum response when negative phase is included.

#### Different Yielding Strength ( $F_y$ ).

The MATLAB programme was used to analyse the given structural element using different parameters: To values of  $F_y$  namely  $F_y = 100\text{MPa}$  and length to thickness ratio equal to 50.

#### When Yielding Strength ( $F_y = 100\text{ MPa}$ ) & ( $\frac{l}{H} = 50$ ).

Analysis have done by using Math lab programme, Figure (11) shows two blast events and the structural response in each stage, and a reduction of structural response that is resulted from negative phase. The red line gives an indication that the structural deformation reach the plastic zone, where the resistance of the element was assumed to be constant.



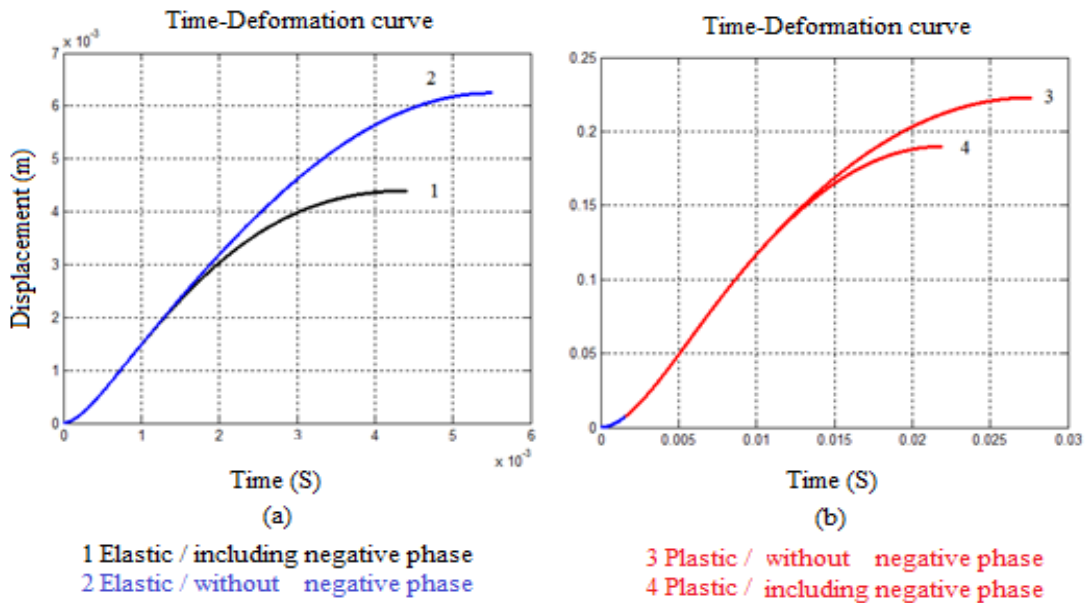


Figure 11: Negative phase effect on structural response ( $L/H=50$ ,  $F_Y=100$  MPa): Charge weight (1 kg), (b) Charge weight (10 kg).

The MATLAB programme was used to analyse the given structural element using different parameters: To values of  $F_y$  namely  $F_y = 400$  MPa and length to thickness ratio equal to 50.

When Yielding Strength ( $F_y = 400$  MPa) & ( $\frac{L}{H} = 50$ ).

In this case, the yielding strength was changed, and measuring the structural response due to two blast events in each stage as Figure (12) illustrated.

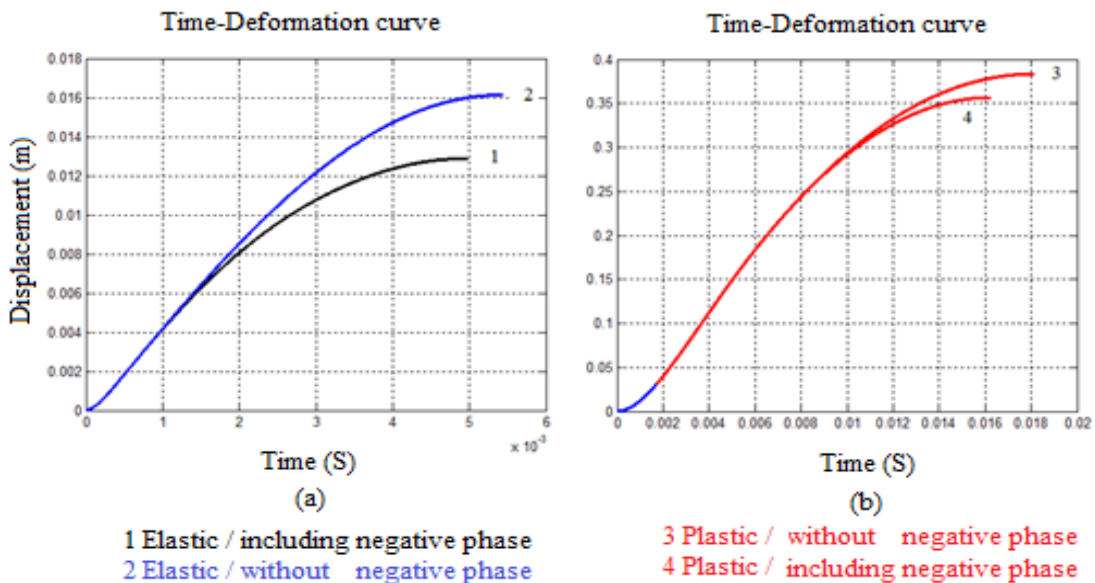


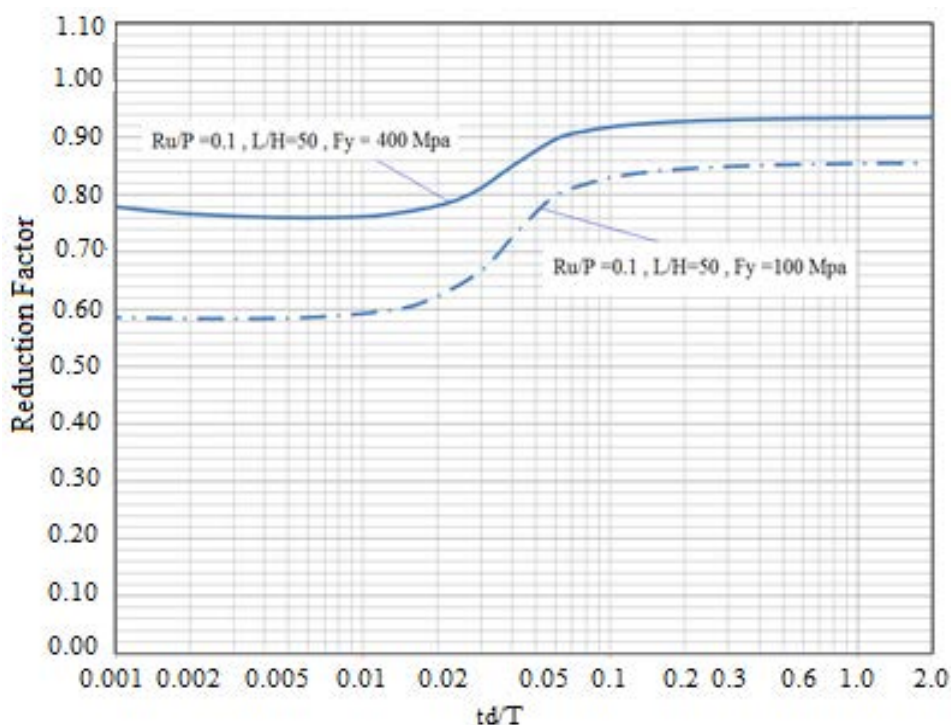
Figure 12: Negative phase effect on structure response ( $L/H = 50$ ,  $F_Y = 400$  MPa): Charge weight (1 kg), (b) Charge weight (10 kg).

It can be clearly seen from the two cases above is that the negative phase has an effective contribution in reducing of structure response. Table (4.1) shows the structural response result for both cases.

**Table 1: Structure response when the yielding strength (FY) different.**

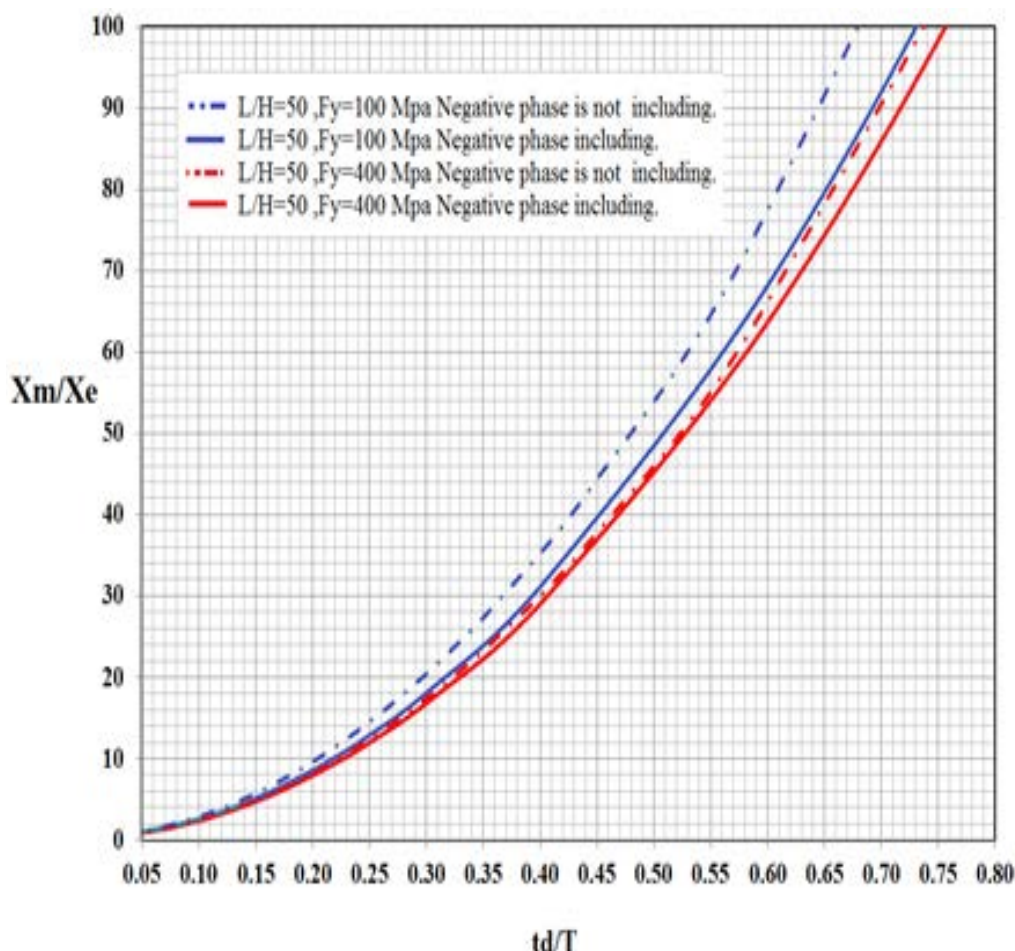
	Figure (4-1-a)	Figure (4-1-b)	Figure(4-2-a)	Figure (4-2-b)
	Yielding strength (Fy = 100 MPa) (L/H = 50)		Yielding strength (Fy = 400 MPa) (L/H = 50)	
Spherical charge weight	1 (kg)	10 (kg)	1 (kg)	10 (kg)
Scaled distance (z). $(\frac{m}{kg^{1/3}})$	1.86	1.86	1.16	1.16
Distance (S). (m)	1.86	4.02	1.16	2.50
$\frac{td}{T}$	0.0358	0.3576	0.0243	0.2433
Reduction factor (R.F)	0.7041	0.8516	0.7982	0.9302
$(\frac{x_m}{x_e}) - ve$	0.5628	24.2967	0.4121	11.4116
$(\frac{x_m}{x_e}) + ve$	0.7993	28.5291	0.5163	12.2678

The difference in each response due to changes in the yielding strength (Fy) can be shown clearly in Figure (13). Both trends shows the reduction factor increases, when (td/T) became smaller. This illustrates the flexible structural has to response effectively to the under-pressure of blast load curve.



**Figure 13: The differences in reduction factor when (L/H = 50) and different yielding strength.**

Furthermore, there are some of key aspects that could be noticed in Figure (14), as result from different yielding strength and influence of negative phase. The figure shows the maximum deflection to elastic deformation ( $X_m/X_e$ ) is less when negative phase is added. In addition, the differences in the deformation between including or not of negative phase in blast load become bigger when the yielding strength is small at the same element dimension.

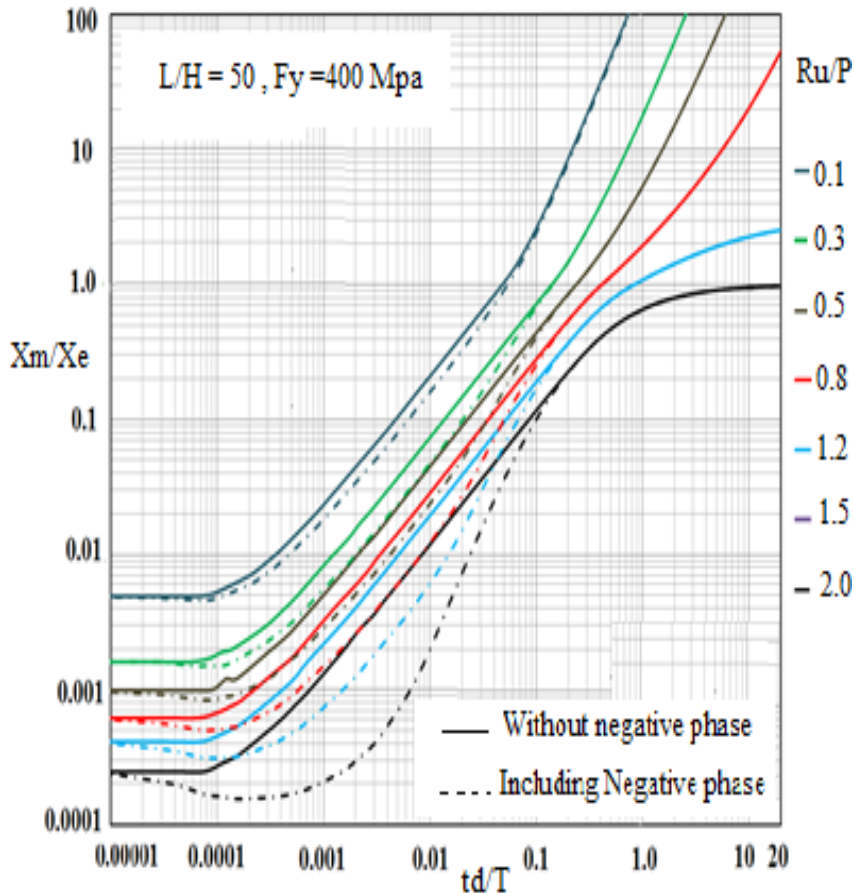


**Figure 14: The differences in maximum deflection to elastic deformation ratio ( $X_m/X_e$ ) when ( $L/H = 50$ ) and different yielding strength.**

To see the general image of negative phase contribution on structural response, figure (15) shows the reduction in the response as a result of including negative phase for different scenarios of blast events for structure has parameters of  $F_y = 400$  MPa and length to thickness ratio equal to 50. In addition, more reduction can be seen when ( $R_u/P$ ) is large. This reason why the structure takes more initially time, when the blast wave hits the structure and start to response. This delay in structure movement enables the time for under pressure period to reach the element and influence on structural response effectively. Theoretically, when the taken time of structure that have been taken to reach the maximum first deformation ( $t_{max}$ ) is more than the duration time ( $t_d$ ) of positive phase of the blast load, then the negative phase has a significant effect in response of the structure. Otherwise, it does not have any effect as first peak deflection is the target. However, the additional duration time from the negative phase is needed for the

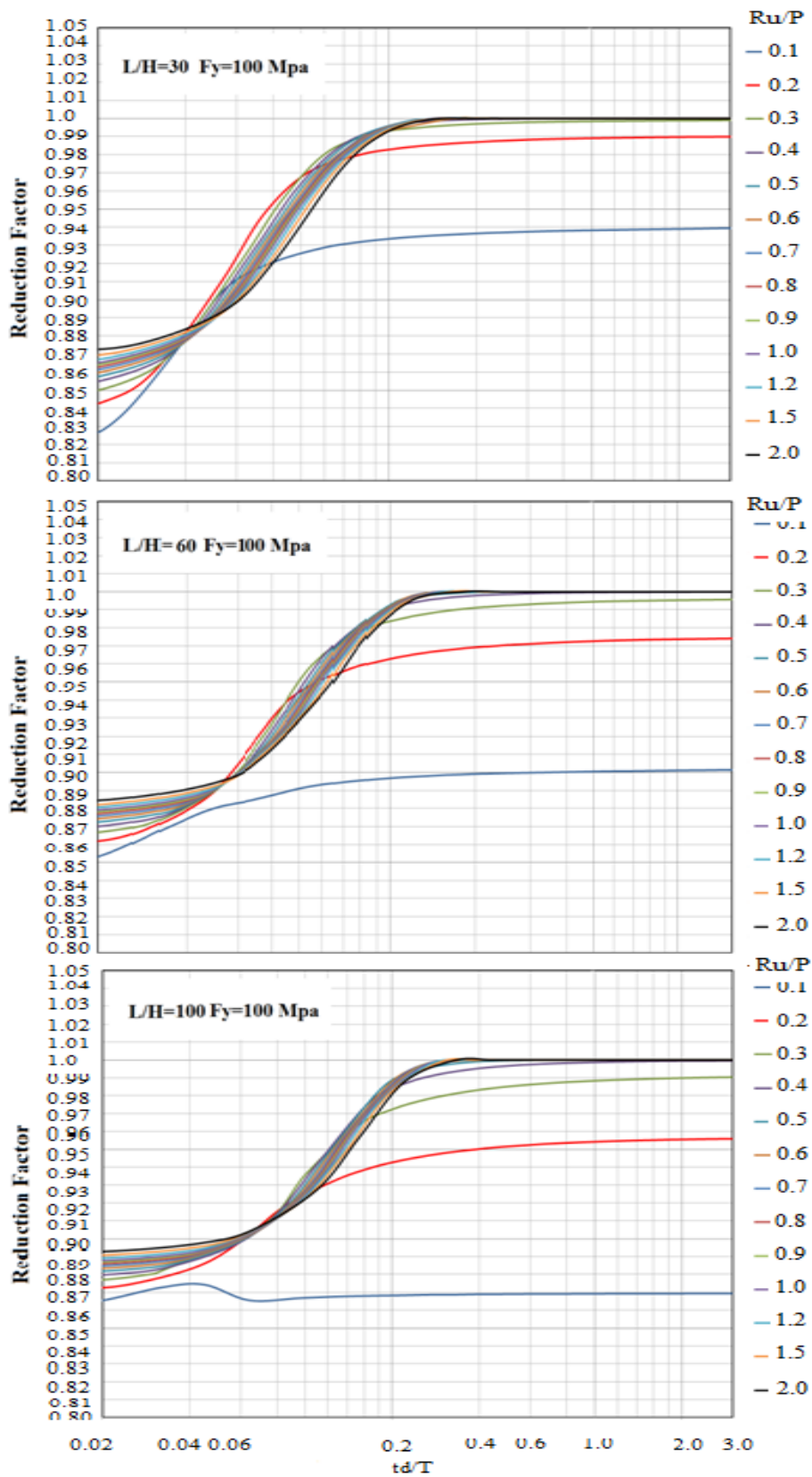
structure to reach to the balance, and then it could be clearly seen the minimization of structural response due to under pressure of negative phase.

The reduction factor for structures with low natural period of time, in other words  $(td/T) > 0.1$  has no effect. This because the time to take for reaching its first maximum deformation is lower than the duration time of positive phase.



**Figure 15: Maximum deflection ( $X_m/X_e$ ) of elastic– plastic SDOF with negative effect of non-linear approximation of blast pressure with different ( $R_u/P$ ).**

It has been approved that the under-pressure has a major contribution on the deformation of flexible structural element that is subjected to impact loading. From the findings that have been gotten, charts with reduction factors could be generated at any different parameters of element geometry or yielding strength. Reduction factors charts in Figures (16 and 17), which can be practically used to determine the reduction in structure response if under-pressure was added to blast pressure time history.



**Figure 16: Reduction factors when ( $F_y = 100$  MPa).**

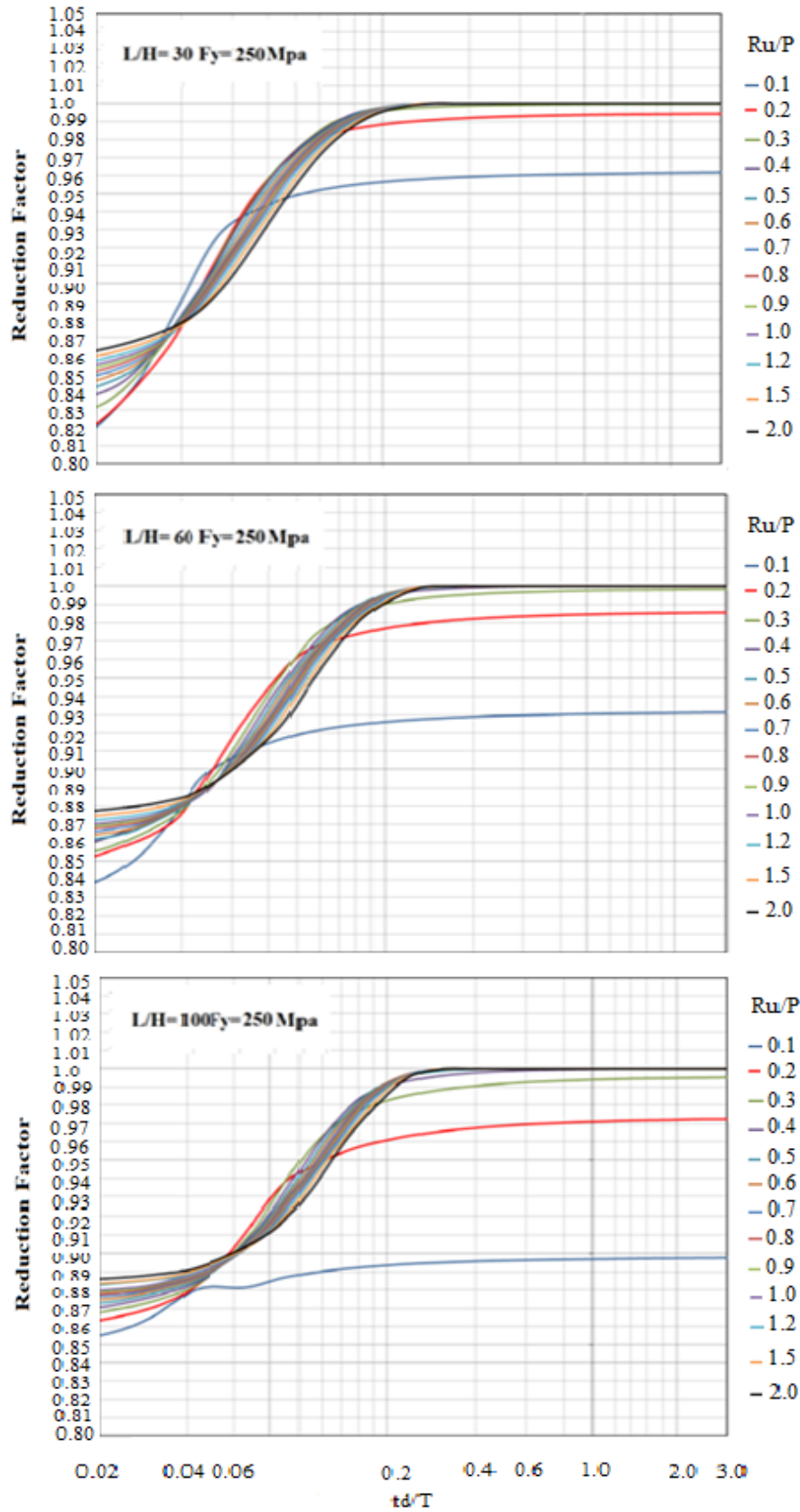


Figure 17: Reduction factors when ( $F_y = 250\text{MPa}$ ).

## CONCLUSIONS

Negative phase of blast load curve has a reduction influence on the response of flexible structure. Therefore, it is important to include the under-pressure part of blast load in the load function during response analysis. Even though adding negative phase will decrease the ultimate deformation of the structural element, and could be considered conservative. However, the reduction will be economical in the member design to withstand against blast impact. In this paper, it has been proved that including under-pressure will cause a minimization of maximum deformation of flexible structure, or when the time duration of positive phase is less than the time for structure member to reach its first peak deflection. From this finding, more accurate and specified charts for maximum deformation prediction can be generate for engineers use.

## FURTHER WORKS

It is recommended that doing further works, in order to improve the accuracy of the effect of negative phase on structural response in future researches. The limitation of this work is the assumption that has been made for structural response behaviour for the SDOF system. It can be recommended to increase the complicity of resistance-deformation function by using the accurate spectra where the stiffness is changeable with deformation instead of elastic-purely plastic assumption to gain more realistic result of structural response. In addition, having more understanding on how modelling blast load with more than one shock wave.

## NOTATIONS

$T$ : Natural period tome of structure (sec)

$K_m$ ,  $K_L$  and  $K_r$ : Equivalent factor of mass, load and resistance.

$R_{max}$ ,  $R_u$ : Ultimate resistance of the structure (N).

$X_e$ ,  $X_m$ : Elastic and maximum deformation (m)

$R_{rmax}$ ,  $R_{rmin}$ : Overpressure and under pressure (KPa)

$td$ : Duration time of positive phase for non-linear approximation(sec).

$tdn$ : Duration time of negative phase for non-linear approximation(sec).

$c$ : Coefficient decay.

$Z$ : Scaled distance ( $m/kg^{1/3}$ ).

$S$ : Distance (m)

$T_m$ : Time of maximum deformation.

$L/H$ : Length height ratio of structural member.

$F_y$ : Yielding strength (MPa).

$\ddot{Z}$ : Acceleration ( $m/s^2$ ).

$\dot{Z}$ : Velocity (m/s).

$z$ : Displacement (m).

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