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Transient Solution of an M/M/2 Queue with Feedback, Catastrophe, and Repair

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Abstract

This paper presents a transient solution that is obtained analytically through processing the probability-generating function using Laplace transforms independent upon the theory of complex analysis to Rauch regarding the system size in an M/M/2 feedback queue with the possibility of catastrophes at services failures and repairs. Asymptotic behavior of average queue length is deduced. Further numerical illustrations have been used to discuss the asymptotic behavior of the mean system size.

Keywords: Transient analysis, feedback, queueing system, catastrophe, repair, generating function, asymptotic behavior.

Introduction

In the study of queuing systems, the determination of transient solution is very much essential to analyze the behavior of the system. Transient analysis is very useful for all queuing models to obtain optimal solutions which pave the way to control the system *(1)*. Among several methods, probability-generating function using Laplace transforms dependent on the theory of complex analysis in Rauch is one of the techniques that used to obtain a transient solution. Even in the case of a simple M/M/2 queue, an analytical approach to obtain transient behavior is very difficult *(1)*. In this regard, it has obtained the transient solution for an M/M/2 feedback queue which is subject to catastrophes and repairs by employing an effective probability-generating function technique which is very simple. In particular, queues with feedback occur in production systems subject to rework, computer networks, telecommunication systems, Supermarkets, banking industries, hospital management, etc *(1)*. Several authors have investigated queuing systems subject to feedback. Thangaraj and Vanitha *(2)* have studied the Analysis of M/M/1 Queue with Feedback and Catastrophe using Continued Fractions. Another study also introduced a transient Solutions of M/M/1 queue with catastrophes *(3)*. Queuing systems with catastrophes have been studied *(4-6)*. Other studies have focused to study queuing systems with feedback and catastrophes *(2, 7)*. Queuing systems with feedback, catastrophes, and repair have also studied *(8-11)*. Finally, Amin and Venkatesan Focused on SPC techniques using M/M/2 queuing model *(12)*.

In this article, it has proposed transient behavior for the non-truncated two-channel Markovian queue: M/M/2 subject to feedback, catastrophe, and repair. The probability that there are n customers in the system, the probability of empty system, and the asymptotic behavior of the average queue length is obtained using the probability generating function, Laplace and the inverse Laplace transforms. Some special cases are deduced. Finally, a simulation study has been considered to illustrate the numerical application for the asymptotic behavior of the average queue length.

1. Basic Notations and Assumptions

To construct the system of this paper, by defining the following parameters:

$P(s,t) =$Probability generating function.

$P^*(s, z) =$ Laplace transform of $P(s,t)$.

$p_n(t) =$Transient state probability that there are exactly *n* customers in the system.

$p^*_n(z) =$ Laplace transform of $p_n(t)$.

$p_0(t) =$Probability that no customers are in the service department at time *t*.

$p^*_0(z) =$ Laplace transform of $p_0(t)$.

$Q(t) =$Probability that the server is under repair at time *t*.

$Q^*(z) =$ Laplace transform of $Q(t)$.

$p_n =$Stead-state probability that there are *n* customers in the system.

$\lambda =$ Mean arrival rate.

$\mu =$ Mean service rate per service representative.

$q =$Probability that a customer joins the departure process.

$1-q =$Probability that a customer joins the end of the original queue.

$\upsilon =$Catastrophe rate.

$\eta =$ Repair rate.

n = Number of customers in the system.

$\rho = \lambda/2\mu$ = Utilization factor.

$m(t)$ = Time-dependent expected value.

$m^*(s)$ = Laplace transform of $m(t)$.

The assumptions of this model are listed as follows:

(1) Customers arrive at the server one by one according to the Poisson process with rate $\lambda_n = \lambda$.

(2) Service times of the customers are independent and identically distributed *(iid)* exponential random variables with rate μ_n . The customers are served according to FCFS discipline.

(3) After completion of each service the customer either joins at the end of the original queue as a feedback customer with the probability $(1-q)$ or departure the system with probability q.

(4) The catastrophe occurs at the service department according to the Poisson process with rate υ when the system is not empty or empty. The occurrence of a catastrophe destroys all the customers in the instants and affects the system as well.

(5) The repair times of the failed server after a catastrophe is *iid* exponential random variables with rate η. After a repair on the server is completed, the server immediately returns to its working position for service when a new customer arrives.

1. Model Formulation and Analysis

From the above notations and assumptions and applying Markov conditions, it was obtaining the following probability differential-difference equations as:

$$Q'(t) = -(\eta+\upsilon)\,Q(t)+\upsilon \qquad [1[$$

$$p_0'(t) = -(\lambda+\upsilon)\,p_0(t)+q\,\mu\,p_1(t)+\eta\,Q(t), \qquad n=0 \qquad [2]$$

$$p_1'(t) = \lambda\,p_0(t)-(\lambda+q\mu+\upsilon)\,p_1(t)+2q\,\mu\,p_2(t), \qquad n=1 \qquad [3]$$

$$p_n'(t) = \lambda\,p_{n-1}(t)-(\lambda+2q\,\mu+\upsilon)p_n(t) + 2q\,\mu\,p_{n+1}(t), \qquad n\geq 2 \qquad [4]$$

Taking Laplace transform on equations [1] and [3], then get:

$$Q^*(z) = \frac{\upsilon}{z(z+\eta+\upsilon)} \qquad [5]$$

$$p_1^*(z) = \frac{\lambda p_0^*(z)}{(z+\lambda+q\mu+\upsilon)} + \frac{2q\mu\, p_2^*(z)}{(z+\lambda+q\mu+\upsilon)} \qquad [6]$$

By Inverse $Q^*(z)$, $p_1^*(z)$ in equations [4] and [5], to get the explicit expression for $Q(t)$, $p_1(t)$ as:

$$Q(t) = \frac{\upsilon}{(\eta+\upsilon)}\left(1-e^{-(\eta+\upsilon)t}\right), \qquad [7]$$

$$p_1(t) = \lambda\int_0^t p_0(u)e^{-(\lambda+\mu q+\upsilon)(t-u)}du$$

$$+2\mu q\int_0^t p_2(u)e^{-(\lambda+\mu q+\upsilon)(t-u)}du \qquad [8]$$

Consider the probability generating function as:

$$P(s,t) = \sum_{n=0}^{\infty} p_n(t)s^n,\ P(s,0) = 1 \text{ and } Q(0) = 0 \qquad [9]$$

Differentiating [9] and using equation [4], it obtain:

$$\frac{\partial P(s,t)}{\partial t} = \frac{1}{s}\left\{\left[\lambda s^2 + 2q\mu - (\lambda + 2q\mu + \upsilon)s\right]p(s,t)\right.$$

$$\left. -q\mu(1-s)sp_1(t) - q\mu(1-s)p_0(t) + \eta sQ(t)\right\} \qquad [10]$$

The solution of the above differential equation is easily obtained as:

$$P^*(s,z) = \frac{s^2 - q\mu(1-s)sp_1^*(z) - 2q\mu(1-s)p_0^*(z) - +\eta sQ(t)}{(z+\lambda+2q\mu+\upsilon)s - 2q\mu - \lambda s^2},$$

$$|s| \le 1, Re(z) > 0 \qquad [11]$$

with $p_0^*(z) = \dfrac{s_1^2 - q\mu(1-s_1)s_1 p_1^*(z) + \eta s_1 Q(z)}{2q\mu(1-s_1)}$ [12]

From equations [11] and [12], to find:

$$P^*(s,z) = \frac{1}{\lambda s_2}\left\{\left[s + s_1 - q\mu(1-s)p_1^*(z)\right]\sum_{i=0}^{\infty}(s/s_2)^i\right.$$

$$+\left(s_1^2+\eta Q^*(z)\right)\left(\frac{1}{1-s_1}\right)\sum_{i=0}^{\infty}\left(s/s_2\right)^i\Bigg\}, \qquad \left(\left|s/s_2\right|<1\right) \qquad [13]$$

Substituting $P^*(s,z)=\sum_{n=0}^{\infty}p_n^*(z)s^n$, and comparing the coefficients of s^n on both sides, and some algebra, it find:

$$p_n^*(z)=\frac{1}{\lambda}\left\{\frac{1}{s_2^n}+\frac{2q\mu/\lambda}{s_2^{n+2}}-q\mu\left(\frac{1}{s_2^{n+1}}\quad\frac{1}{s_2^n}\right)p_1^*(z)+\left(\frac{\lambda}{2q\mu}\right)^{n+1}\right.$$

$$\left.\times\left[\sum_{j=n+3}^{\infty}\left(\frac{2q\mu}{\lambda s_2}\right)^j+\eta Q^*(x)\sum_{j=n+1}^{\infty}\left(\frac{2q\mu}{\lambda s_2}\right)^j\right]\right\}, \; n\geq 2 \qquad [14]$$

Substituting equations [5] and [6] into [14] and taking the inverse Laplace to transform by using some properties of Bessel functions, we gained $p_n(t)$ explicitly of $\lambda,\mu,\ q,\upsilon,\eta$ and *t* as:

$$p_n(t)=\frac{e^{-(\lambda+2q\mu+\upsilon)t}}{\lambda t}\left\{\left(\frac{\lambda}{2q\mu}\right)^{\frac{n}{2}}\left[(n+2)I_{n+2}\left(2\sqrt{2\lambda q\mu}\,t\right)+nI_n\left(2\sqrt{2\lambda q\mu}\,t\right)\right]\right.$$

$$-q\mu\left(\lambda\int_0^t p_0(u)e^{-(\lambda+q\mu+\upsilon)(t-u)}du+2q\mu\int_0^t p_2(u)e^{-(\lambda+q\mu+\upsilon)(t-u)}du\right)$$

$$\times\left(\left(\frac{\lambda}{2q\mu}\right)^{\frac{n+1}{2}}(n+1)I_{n+1}\left(2\sqrt{2\lambda q\mu}\,t\right)-\left(\frac{\lambda}{2q\mu}\right)^{\frac{n}{2}}nI_n\left(2\sqrt{2\lambda q\mu}\,t\right)\right)$$

$$+\sum_{j=n+3}^{\infty}j\left(\frac{2q\mu}{\lambda}\right)^{\frac{j+2(n+1)}{2}}I_j\left(2\sqrt{2\lambda q\mu}\,t\right)$$

$$+\frac{\eta\upsilon}{(\eta+\upsilon)}\sum_{j=n+1}^{\infty} j\left(\frac{2q\mu}{\lambda}\right)^{\frac{j+2(n+1)}{2}} I_j\left(2\sqrt{2\lambda q\mu t}\right)\Bigg\},\quad n\geq 2 \quad [15]$$

From equations [5], [6] and [12], to get:

$$p_0^*(z)=\frac{1}{2q\mu}\left[\sum_{j=2}^{\infty}\left(\frac{2q\mu}{\lambda}\right)^{\frac{j}{2}}\left(\frac{2\sqrt{2\lambda q\mu}}{z+\lambda+2q\mu+\upsilon+\sqrt{(z+\lambda+2q\mu+\upsilon)^2-8\lambda q\mu}}\right)^j\right.$$

$$-\left(\frac{q\mu}{2\lambda}\right)^{\frac{1}{2}}\left(\frac{2\sqrt{2\lambda q\mu}}{\theta+\sqrt{\theta^2-8\lambda q\mu}}\right)\left(\frac{\lambda p_0^*(z)}{(z+\lambda+q\mu+\upsilon)}\right.$$

$$\left.+\frac{2q\mu\, p_2^*(z)}{(z+\lambda+q\mu+\upsilon)}\right)+\frac{\eta\upsilon}{z(z+\eta+\upsilon)}$$

$$\left.\times\sum_{j=1}^{\infty}\left(\frac{2q\mu}{\lambda}\right)^{\frac{j}{2}}\left(\frac{2\sqrt{2\lambda q\mu}}{z+\lambda+2q\mu+\upsilon+\sqrt{(z+\lambda+2q\mu+\upsilon)^2-8\lambda q\mu}}\right)^j\right] \quad [16]$$

Which on inversion yields:

$$p_0(t)=\frac{e^{-(\lambda+2q\mu+\upsilon)t}}{2q\mu t}\left[\sum_{j=2}^{\infty} j\left(\frac{2q\mu}{\lambda}\right)^{j/2} I_j\left(2\sqrt{2\lambda q\mu}\,t\right)\right.$$

$$+\frac{\eta\upsilon}{(\eta+\upsilon)}\sum_{j=2}^{\infty} j\left(\frac{2q\mu}{\lambda}\right)^{j/2} I_j\left(2\sqrt{2\lambda q\mu}\,t\right)-q\mu\left(\frac{2q\mu}{\lambda}\right)^{1/2}$$

$$\times I_1\left(2\sqrt{2\lambda q\mu}\,t\right)\left(\lambda\int_0^t p_0(u)e^{-(\lambda+q\mu+\upsilon)(t-u)}du\right.$$

$$\left.\left.+2q\mu\int_0^t p_2(u)e^{-(\lambda+q\mu+\upsilon)(t-u)}du\right)\right] \quad [17]$$

2. Asymptotic Behaviour of Average Queue Length

In this section, it was derived from the asymptotic behavior of the mean system size $m(t)$. Define the average queue length $m(t)$ as follows:

$$m(t)=E(n)=\sum_{n=1}^{\infty} n\, p_n(t)=\frac{d\,P(s,t)}{d\,s}\downarrow_{s=1} \qquad [18]$$

From equation [18], differentiating the equation [10] with respect to *z* and evaluating at *z= 1*, obtained as:

$$\frac{dm(t)}{dt}+\upsilon\, m(t)=\lambda-2q\mu+2q\mu\, p_0(t)+q\mu\, p_1(t)+(q\mu-\lambda)Q(t) \quad [19]$$

Solving the differential equation for $m(t)$ with $m(0)=\sum_{n=1}^{\infty} np_n(t)=0$, to be:

$$m(t)=(\lambda-2q\mu)\int_0^t e^{-\upsilon(t-u)}du+2q\mu\int_0^t p_0(u)e^{-\upsilon(t-u)}du$$

$$+q\mu\int_0^t p_1(t)e^{-\upsilon(t-u)}du+(q\mu-\lambda)\int_0^t Q(t)e^{-\upsilon(t-u)}du \qquad [20]$$

Taking the Laplace to transform on both sides of equation [20], using final value tauberian theorem $\lim_{t\to\infty} m(t)=\lim_{s\to 0} sm^*(s)$ [15], we obtain $m(t)$ explicitly of λ,μ,q,η and υ as:

$$m(t)=\frac{\lambda-2q\mu}{\upsilon}+\frac{1}{(\eta+\upsilon)}\left\{q\mu-\lambda+2\eta\, q\mu\right.$$

$$\left.\times\sum_{j=1}^{\infty}\left(\frac{4q\mu}{\lambda+2q\mu+\upsilon+\sqrt{(\lambda+2q\mu+\upsilon)^2-8\lambda\, q\mu}}\right)^j\right\},\quad t\to\infty \qquad [21]$$

3. Cases Special

Some queuing systems can be obtained as special cases of this system:

Case one: let $q=1$, this is the queue: M/M/2 with catastrophe and repair. Then relations [15], [17] and [21] are given by:

The probability that there are *n* customers in the system at time *t* is:

$$p_n(t)=\frac{e^{-(\lambda+2\mu+\upsilon)t}}{\lambda t}\left\{\left(\frac{\lambda}{2\mu}\right)^{\frac{n}{2}}\left[(n+2)I_{n+2}\left(2\sqrt{2\lambda\mu}\,t\right)+n\,I_n\left(2\sqrt{2\lambda\,\mu}\,t\right)\right]\right.$$

$$-\mu\left(\lambda\int_0^t p_0(u)e^{-(\lambda+\mu+\upsilon)(t-u)}du+2\mu\int_0^t p_2(u)e^{-(\lambda+\mu+\upsilon)(t-u)}du\right)$$
$$\times\left(\left(\frac{\lambda}{2\mu}\right)^{\frac{n+1}{2}}(n+1)I_{n+1}\left(2\sqrt{2\lambda\mu}\,t\right)-\left(\frac{\lambda}{2\mu}\right)^{\frac{n}{2}}nI_n\left(2\sqrt{2\lambda\mu}\,t\right)\right)$$
$$+\sum_{j=n+3}^{\infty}j\left(\frac{2\mu}{\lambda}\right)^{\frac{j+2(n+1)}{2}}I_j\left(2\sqrt{2\lambda\mu}\,t\right)+\frac{\eta\upsilon}{(\eta+\upsilon)}$$
$$\left.\times\sum_{j=n+1}^{\infty}j\left(\frac{2\mu}{\lambda}\right)^{\frac{j+2(n+1)}{2}}I_j\left(2\sqrt{2\lambda\mu}\,t\right)\right\},\ n\geq 2 \qquad [22]$$

The probability that no customers in the system department is:

$$p_0(t)=\frac{e^{-(\lambda+2\mu+\upsilon)t}}{2\mu t}\left[\sum_{j=2}^{\infty}j\left(\frac{2\mu}{\lambda}\right)^{j/2}I_j\left(2\sqrt{2\lambda\mu}\,t\right)\right.$$
$$+\frac{\eta\upsilon}{(\eta+\upsilon)}\sum_{j=2}^{\infty}j\left(\frac{2\mu}{\lambda}\right)^{j/2}I_j\left(2\sqrt{2\lambda\mu}\,t\right)-\mu\left(\frac{2\mu}{\lambda}\right)^{\frac{1}{2}}$$
$$\times I_1\left(2\sqrt{2\lambda\mu}\,t\right)\left(\lambda\int_0^t p_0(u)e^{-(\lambda+\mu+\upsilon)(t-u)}du\right.$$
$$\left.\left.+2\mu\int_0^t p_2(u)e^{-(\lambda+\mu+\upsilon)(t-u)}du\right)\right], \qquad [23]$$

The average queue length of the system is

$$m(t)=\frac{\lambda-2\mu}{\upsilon}+\frac{1}{(\eta+\upsilon)}\left\{\mu-\lambda+2\mu\eta\right.$$
$$\left.\times\sum_{j=1}^{\infty}\left(\frac{4\mu}{\lambda+2\mu+\upsilon+\sqrt{(\lambda+2\mu+\upsilon)^2-8\lambda\mu}}\right)^j\right\},\quad t\to\infty \quad [24]$$

Relations [22], [23] and [24] are the same results as Ammar (*8*) if $\mu_1 = \mu_2 = \mu$.

Case two: let $\eta = 0$, this is the queue: M/M/2 with feedback and catastrophe. Then relations [7], [15], [17]and [21] are given by:

The probability that the server is under repair at time *t* is:

$$Q(t) = 1 - e^{-\upsilon t}, \qquad [25]$$

The probability that there are *n* customers in the system at time *t* is:

$$p_n(t) = \frac{e^{-(\lambda+2q\mu+\upsilon)t}}{\lambda t}\Bigg\{\left(\frac{\lambda}{2q\mu}\right)^{\frac{n}{2}}\left[(n+2)I_{n+2}\left(2\sqrt{2\lambda q\mu}\,t\right) + n\,I_n\left(2\sqrt{2\lambda q\mu}\,t\right)\right]$$

$$-q\mu\left(\lambda\int_0^t p_0(u)e^{-(\lambda+q\mu+\upsilon)(t-u)}du + 2q\mu\int_0^t p_2(u)e^{-(\lambda+q\mu+\upsilon)(t-u)}du\right)$$

$$\times\left(\left(\frac{\lambda}{2q\mu}\right)^{\frac{n+1}{2}}(n+1)I_{n+1}\left(2\sqrt{2\lambda q\mu t}\right) - \left(\frac{\lambda}{2q\mu}\right)^{\frac{n}{2}} nI_n\left(2\sqrt{2\lambda q\mu}\,t\right)\right)$$

$$+\sum_{j=n+3}^{\infty} j\left(\frac{2q\mu}{\lambda}\right)^{\frac{j+2(n+1)}{2}} I_j\left(2\sqrt{2\lambda q\mu}\,t\right)\Bigg\}, \qquad n \ge 2 \qquad [26]$$

The probability that no customers in the system department is:

$$p_0(t) = \frac{e^{-(\lambda+2q\mu+\upsilon)t}}{2q\mu t}\left[\sum_{j=2}^{\infty} j\left(\frac{2q\mu}{\lambda}\right)^{j/2} I_j\left(2\sqrt{2\lambda q\mu}\,t\right)\right], \qquad [27]$$

The average queue length of the system is

$$m(t) = \frac{\lambda - 2q\mu}{\upsilon} + \frac{q\mu - \lambda}{\upsilon} \quad , \quad t \to \infty \qquad [28]$$

Relations [25], [26], [27] and [28] are the same results as Shanmugasundaram and Chitra (*7*).

Case three: let $q = 1$ and $\eta = 0$, this is the queue: M/M/2 with catastrophe. Then relations [7], [15], [17] and [21] are given by:

The probability that the server is under repair at time *t* is:

$$Q(t) = 1 - e^{-\upsilon t}, \qquad [29]$$

The probability that there are *n* customers in the system at time *t* is:

$$p_n(t)=\frac{e^{-(\lambda+2\mu+\upsilon)t}}{\lambda t}\left\{\left(\frac{\lambda}{2\mu}\right)^{\frac{n}{2}}\left[(n+2)I_{n+2}\left(2\sqrt{2\lambda\mu}\,t\right)+n\,I_n\left(2\sqrt{2\lambda\mu}\,t\right)\right]\right.$$

$$-\mu\left(\lambda\int_0^t p_0(u)e^{-(\lambda+\mu+\upsilon)(t-u)}du+2\mu\int_0^t p_2(u)e^{-(\lambda+\mu+\upsilon)(t-u)}du\right)$$

$$\times\left(\left(\frac{\lambda}{2\mu}\right)^{\frac{n+1}{2}}(n+1)I_{n+1}\left(2\sqrt{2\lambda\mu}\,t\right)-\left(\frac{\lambda}{2\mu}\right)^{\frac{n}{2}}nI_n\left(2\sqrt{2\lambda\mu}\,t\right)\right)$$

$$\left.+\sum_{j=n+3}^{\infty}j\left(\frac{2\mu}{\lambda}\right)^{\frac{j+2(n+1)}{2}}I_j\left(2\sqrt{2\lambda\mu}\,t\right)\right\},\qquad n\geq 2 \qquad [30]$$

The probability that no customers in the system department is:

$$p_0(t)=\frac{e^{-(\lambda+2\mu+\upsilon)t}}{2\mu t}\left[\sum_{j=2}^{\infty}j\left(\frac{2\mu}{\lambda}\right)^{j/2}I_j\left(2\sqrt{2\lambda\mu}\,t\right)\right], \qquad [31]$$

The average queue length of the system is

$$m(t)=\frac{\lambda-2\mu}{\upsilon}+\frac{\mu-\lambda}{\upsilon}\quad , \quad t\to\infty \qquad [32]$$

Relations [29], [30], [31] and [32] are the same results as Kumar, *et al.*,(*4*) if $\mu_1=\mu_2=\mu$.

Case four: let $q=1$ and $\upsilon=0$, this is the queue: M/M/2 without any concepts. Then relations [7], [15] and [17] are given by:

The probability that the server is under repair at time *t* is:

$$Q(t)=0, \qquad [33]$$

The probability that there are *n* customers in the system at time *t* is:

$$p_n(t)=\frac{e^{-(\lambda+2\mu)t}}{\lambda t}\left\{\left(\frac{\lambda}{2\mu}\right)^{\frac{n}{2}}\left[(n+2)I_{n+2}\left(2\sqrt{2\lambda\mu}\,t\right)+n\,I_n\left(2\sqrt{2\lambda\mu}\,t\right)\right]\right.$$

$$-\mu\left(\lambda\int_0^t p_0(u)e^{-(\lambda+\mu)(t-u)}du+2\mu\int_0^t p_2(u)e^{-(\lambda+\mu)(t-u)}du\right)$$

$$\times\left\{\left(\left(\frac{\lambda}{2\mu}\right)^{\frac{n+1}{2}}(n+1)I_{n+1}\left(2\sqrt{2\lambda\mu}\,t\right)-\left(\frac{\lambda}{2\mu}\right)^{\frac{n}{2}}nI_n\left(2\sqrt{2\lambda\mu}\,t\right)\right)\right.$$

$$\left.+\sum_{j=n+3}^{\infty}j\left(\frac{2\mu}{\lambda}\right)^{\frac{j+2(n+1)}{2}}I_j\left(2\sqrt{2\lambda\mu}\,t\right)\right\},\qquad n\geq 2 \qquad [34]$$

The probability that no customers in the system department is:

$$p_0(t)=\frac{e^{-(\lambda+2\mu)\,t}}{2\mu t}\left[\sum_{j=2}^{\infty}j\left(\frac{2\mu}{\lambda}\right)^{j/2}I_j\left(2\sqrt{2\lambda\mu}\,t\right)\right], \qquad [35]$$

Relations [33], [34] and [35] are the same results as Groos and Harris (*13*) if $\mu_1=\mu_2=\mu$.

4. An Illustrative Example

The result of $m(t)$ for different values of λ,μ,υ,q and η are shown in the following Table 1:

Table (1): Computed values of average queue length of the system *m(t)* for varying values of parameters $(\lambda,\mu,\upsilon,q\&\eta)$

λ	μ	N	η	q	$m(t)$
1	2	0.2	0.5	0.2	0.46
2	3	0.4	1.5	0.3	0.78
3	4	0.6	2.5	0.4	0.94
4	5	0.8	3.5	0.5	1.45
5	6	1	4.5	0.6	2.10
6	7	1.2	5.5	0.7	3.15
7	8	1.4	6.5	0.8	4.56

The solution of the model may be determined more readily plotting $m(t)$ is drawn against λ,μ,q,υ and η as shown in Figure 1.

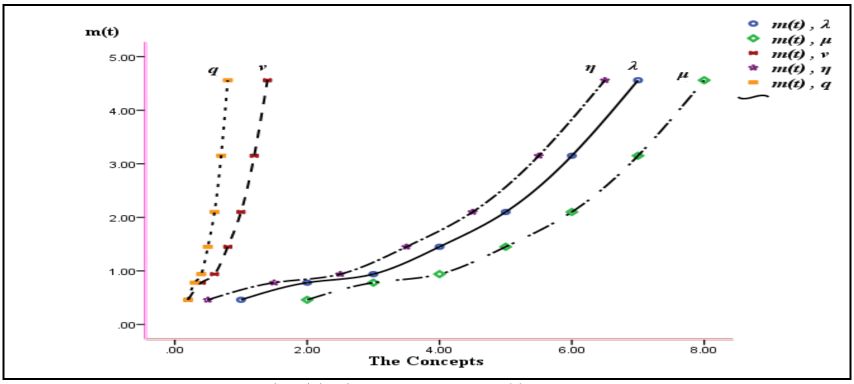

Figure (1) . The relation $(m(t),(\lambda,\mu,q,\upsilon \& \eta))$

As shown in Figure 1, each increase in (arrival rate, service rate, feedback strategy, catastrophe rate, repair) is offset by an increase in the average queue length of the system.

5. Conclusion

The aim of this paper was to obtain the transient solution of the M/M/2 queue with feedback strategy under the impact of both the catastrophe and repair. The transient state probabilities, asymptotic behavior of the mean system size, some special cases were obtained. Moreover, the numerical example illustrates that under asymptotic behavior an increase in (arrival rate, service rate, feedback strategy, catastrophe rate, and repair) is offset by an increase in the average queue length of the system.

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تحليل عابر للطابور *M/M/2* مع الغدية المرتدة و الكارثة والاصلاح

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الهدف من هذه الورقة هو استنباط الحل في حالة الزمن للطابور M/M/2 الواقع تحت تأثير التغذية المرتدة والاصلاح عند وقوع الكارثة. مستخدمين في ذلك الدالة المولدة للاحتمال وتحويلات لابلاس بالاعتماد على نظرية Rauch في التحليل المركب. نحصل على استنتاج السلوك المقارب لمتوسط طول قائمة الانتظار. تم اعطاء مثال عددي لتوضيح السلوك المقارب لمتوسط حجم النظام.

الكلمات المفتاحية : عابر للطابور M/M/2 ، التغذية الراجعة ، نظام الطابور ، الكارثة ، الإصلاح ، توليد الوظيفة ، السلوك المقارب .