Effect of Water Demand Allocation and Network Skeletonization on Hydraulic Performance of Water Distribution Networks

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Abstract

The practical modelling of water distribution networks requires a reduction in the number of consumption nodes and connecting pipes in order to save time, computational and data collection efforts. The reduced model called skeletonized network is achieved by aggregating water demands to be normally allocated at pipe intersection nodes while eliminating unimportant pipes from the real network. The proper representation of the real network by the skeletonized model highly depends on the method by which demands are allocated. In this paper, two methods of demand allocation were investigated with respect to the actual situation. The first method uses Thiessen polygons to determine number of consumers to be supplied by each node. The second method uses the principle of insufficient information to distribute demands supplied by each pipe. The hydraulic performance of each method is evaluated using the satisfaction of pressures at the network nodes considering all pipes in service, while measuring the hydraulic performance under pipe failure using the resilience index. The main result of the study is the overestimation of pressures at end nodes of the network when Thiessen method is used. The approximated method outperformed the Thiessen method at all network nodes.

Key Words: water, networks nodes, connecting pipes, Thiessen polygons

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1. Introduction

Real world water distribution networks contain a large number of house connections supplying drinking water to consumers. Such connections are located along pipes and in reality, they represent the demand nodes of the water systems. Clearly, the inclusion of all connections in the hydraulic modelling of water systems is a laborious work that requires a large amount of time, effort and input data in order to model and calibrate the whole system. As a result, the number of demand nodes are normally reduced by eliminating all connections while aggregating and allocating the demands only at the intersection points of pipes. Evidently, how accurate the network model will represent the real network is highly dependent on the method by which the demands are allocated at these intersections. The elimination of consumption points is often accompanied with elimination of some pipes, a process known with network Skeletonization. It should be emphasized that network Skeletonization is constrained by maintaining all consumption nodes be connected to the network. Therefore, any Skeletonization technique must consider ensuring the supply of water to all consumers.

In this paper, two methods of demand allocation are investigated in comparison with a real situation in which the whole consumption connections are maintained as they appear in reality. The first method named Thiessen method, after the development of Thiessen polygons, defines an area around each demand node so that each building inside the boundary of such an area is closer to that node than any of the other nodes. After that, all consumers located within each polygon are supplied with the node of such a polygon. The second method, called approximate method, divides all consumption demands along a pipe equally at the end nodes of the pipe. This can be justified from the principle of insufficient reason that states if there is no reason to believe that one event will occur preferentially compared to another, the event will occur equally likely in any way. This means that each end node of a pipe will be assigned half of the total demand delivered by the pipe.

2. Hydraulic Modelling Equations

The hydraulic modelling of water distribution networks is mainly governed by the equations of conservation of mass and energy. The principle of the conservation of mass gives rise to the continuity equation, while the principle of conservation of energy results in the head loss equation. Therefore, the hydraulic analysis of the water distribution networks can be formulated to be controlled by the following system of nonlinear equations:

$$\sum_{ij \in in(N_j)} Q_{ij} - \sum_{ij \in out(N_j)} Q_{ij} = Q_j \qquad j = 1, \ldots\ldots, N-1 \qquad (1)$$

$$\sum_{ij \in l} h_{ij} = 0 \qquad (2)$$

Where: N is number of nodes; I represents a loop of closed circuit of pipes; Qj is demand or supply at node j; in (NJ) and out (NJ) are all pipe flows to and from node j, respectively. In the analysis, the calculation of head loss (hij) in pipe ij arising from the friction between water and internal pipe walls is dependent on the formula used. Eq. 1 represents the continuity equation, while Eq. 2 represents the head loss equation. In practice, there are three main empirical formulae widely used to calculate the head loss in Eq. 2 (Bhave and Gupta, 2006). These are Hazen–Williams formula, Chezy–Manning formula and Darcy–Weisbach formula given by Eq. 3, Eq. 4 and Eq. 5 respectively as follows:

$$h_{ij} = \eta\, L_{ij} \left(Q_{ij}/C_{ij}\right)^{1.852} D_{ij}^{-4.871} \qquad (3)$$

$$h_{ij} = 10.29 n_{ij}^2 L_{ij} Q_{ij}^2 D_{ij}^{16/3} \qquad (4)$$

$$h_{ij} = \frac{8 f_{ij} L_{ij} Q_{ij}^2}{\pi^2 g D_{ij}} \qquad (5)$$

Where: η is a dimensionless conversion factor (10.67 in SI units); D_{ij}, h_{ij}, L_{ij} and Q_{ij} are respectively diameter in metres, head loss in metres, length in metres and flow rate in cubic metres per second for pipe ij; C_{ij} and n_{ij} are Hazen–Williams roughness coefficient and Manning roughness coefficient respectively; f_{ij} is the coefficient of friction in pipe ij.

The relationship between head loss (h_{ij}) and flow (Q_{ij}) for pipe ij is often expressed in terms of the pipe resistance coefficient (K_{ij}) as follows:

$$h_{ij} = K_{ij} Q_{ij}^{n_f} \tag{6}$$

Where: n_f represents the flow exponent that is equal to 1.852 for Hazen–Williams formula, while it equals 2 for the formulae of Chezy–Manning and Darcy–Weisbach. The resistance coefficients of Hazen–Williams formula, Chezy–Manning formula and Darcy–Weisbach formula can be respectively expressed as:

$$K_{ij} = \frac{\eta L_{ij}}{C_{ij}^{1.852} D_{ij}^{4.871}} \tag{7}$$

$$K_{ij} = \frac{10.29 n_{ij}^2 L_{ij}}{D_{ij}^{16/3}} \tag{8}$$

$$K_{ij} = \frac{8 f_{ij} L_{ij}}{\pi^2 g D_{ij}^5} \tag{9}$$

To assess the capability of the WDS in satisfying nodal demands in full, minimum required heads are normally set at each demand node. This constraint can be expressed as:

$$H_j \geq H_j^{req} \qquad j = 1, \ldots\ldots, N \qquad (10)$$

Where: H_j and H_j^{req} are available and the required head respectively at node *j*. H_j is obtained from the hydraulic simulation, while the required head is the head at a node above which demands are satisfied in full. The introduction of Eq. 10 as one of the governing equations in evaluating the hydraulic performance of the WDS was to ensure there is sufficient pressure at each demand node.

3. Hydraulic Performance Assessment

Since water distribution networks are vulnerable to abnormal conditions where, for example, some pipes are considered unavailable for some period of time due to a number of reasons like pipe repair and pipe replacement, it has been becoming essential to evaluate the hydraulic performance under such conditions. It is well accepted that evaluating real world network performance under all possible pipe failures is an impractical and time−consuming process. As a result, a number of alternative measures of hydraulic performance are developed without the need to simulate any pipe failure. These include, for instance, network entropy (Tanyimboh and Templeman, 1993), resilience index (Todini, 2000), network resilience (Prasad and Park, 2004), surplus power factor (Vaabel et al., 2006), and modified resilience index (Jayaram and Srinivasan, 2008).

The resilience index was adopted herein to assess the hydraulic performance for the water distribution networks due to its simplicity as it requires only nodal heads and nodal demands and no involvement of pipe flows in the calculations is included.

4. Resilience index

The concept of resilience index was introduced by Todini (2000) as a measure of the available surplus power that can be dissipated internally in the event of a failure in water distribution networks. Since the total power in the network is composed of the power dissipated in the pipes and the power delivered to the nodes, Todini (2000) defined the resilience index as:

$$RI = \frac{\sum_{i=1}^{n_n} Q_i^{req}\left(H_i - H_i^{req}\right)}{\sum_{k=1}^{n_r} Q_k H_k + \sum_{j=1}^{n_{pu}} P_j/\gamma - \sum_{i=1}^{n_n} Q_i^{req} H_i^{req}} \tag{11}$$

Where *RI* is resilience index; H_i is available head at node *i* as obtained from hydraulic simulation; H_i^{req} is the required head of node *i* at which the demand is satisfied in full; Q_i^{req} is demand at node i; γ is specific weight of water; Q_k and H_k are respectively outflow and head at reservoir *k*; P_j is power introduced to the network by pump *j*; n_{pu} is number of pumps in the network; n_n is number of demand nodes in the network; and n_r is number of reservoirs in the network.

5. Application Network

The study was applied to a real water distribution network located south-west of the capital city of Tripoli in Libya. The network is a multi-loop system composed of 11 loops, 26 demand nodes, and 39 pipes and supplied by an elevated water tank as shown in Figure 1. The network is located in a flattened area and so all demand and supply nodes are assumed to be at the same elevation. The water tank is actually located some distance from the network but, for simplification, was assumed to be near to the network. The water tank has a capacity of 200 m^3, base height of 15 m, and top height of 25 m. All pipes are made of HDPE material, PE-100 strength class, pressure class of PN-10, nominal pipe diameter of 90 mm, internal pipe diameter of 79.2 mm, and pipe roughness of 0.003 mm. All buildings of the study area are of two-story type and each story accommodated by a single family with an average size of 6 members. The daily water consumption per capita is taken on average at 150 litres with a maximum daily factor of 2.7. The minimum pressure requirement as recommended by the municipality is at 1.2 bar.

Figure 1: Application network

6. Results and Discussion

The network was analysed with three demand configurations based on the allocation method. The first configuration, designated by actual method, uses the actual situation in which all house connections are included in the modelling as shown in Figure 2a. In this case, the demands are allocated at the house connections according to the number of buildings supplied with each connection instead of intersection nodes that are assigned zero demands. The second configuration, named Thiessen method, defines an area around each demand node so that each building inside the boundary of such an area is closer to that node than any of the other nodes. This was generated by the method of Thiessen polygons as shown in Figure 2b.

Figure 2: Allocation methods for water demand aggregation

The last configuration, called approximate method, uses the principle of insufficient reason that states if there is no reason to believe that one event will occur preferentially compared to another, the event will occur equally likely in any way. Applying this principle leads to assigning the total demand supplied by a pipe equally to the end nodes of that pipe. In other words, each end node of a pipe will be assigned half of the total demand delivered by the pipe.

The satisfaction of hydraulic equations for continuity at each node and head losses within each loop was achieved by using the Water GEMS hydraulic modeller (Bentley, 2018). Table 1 shows a comparison of nodal pressures as obtained from the analysis for each method.

Table 1: Nodal pressures

Node No.	Node pressure (bar)		
	Accurate method	Thiessen method	Approximate method
J-1	1.52913	1.52886	1.52886
J-2	1.41216	1.40858	1.40858
J-3	1.38848	1.38478	1.38478
J-4	1.36751	1.38192	1.36467
J-5	1.38674	1.31143	1.38192
J-6	1.31781	1.26491	1.31143
J-7	1.32999	1.25858	1.32363
J-8	1.27968	1.24208	1.26491
J-9	1.27169	1.24204	1.25858
J-10	1.26091	1.23628	1.24208
J-11	1.26042	1.2284	1.24204
J-12	1.31199	1.22656	1.30502
J-13	1.25577	1.22641	1.23628

J−14	1.25003	1.22649	1.2284
J−15	1.24836	1.22478	1.22656
J−16	1.24813	1.22525	1.22641
J−17	1.24824	1.22674	1.22649
J−18	1.24497	1.22521	1.22478
J−19	1.24551	1.22603	1.22525
J−20	1.24833	1.22613	1.22674
J−21	1.2457	1.23237	1.22521
J−22	1.24798	1.22412	1.22603
J−23	1.24594	1.22445	1.22613
J−24	1.25213	1.30502	1.23237
J−25	1.24432	1.32363	1.22412
J−26	1.24464	1.36467	1.22445

Figure 3 shows a comparison of results among the three methods. Clearly, the approximate method outperformed the Thiessen method as it showed close agreement to the accurate method with an average and maximum differences of 1.5% and 2.1% respectively. Even though the Thiessen method showed a close agreement with most of the nodes, the disagreement was significant and it occurred at the end nodes of the network, namely, nodes 24, 25 and 26. The Thiessen method overestimated the nodal pressures especially at node 26 with an increase of pressure at about 12% compared to the accurate method. This increase in pressure is attributed to the underestimation of demands at the end nodes of the network. Evidently, the polygons at the end nodes did not include the house connections in the inner part of the pipes. Accordingly, a careful estimation of demands at end nodes should be considered in case of applying the Thiessen method.

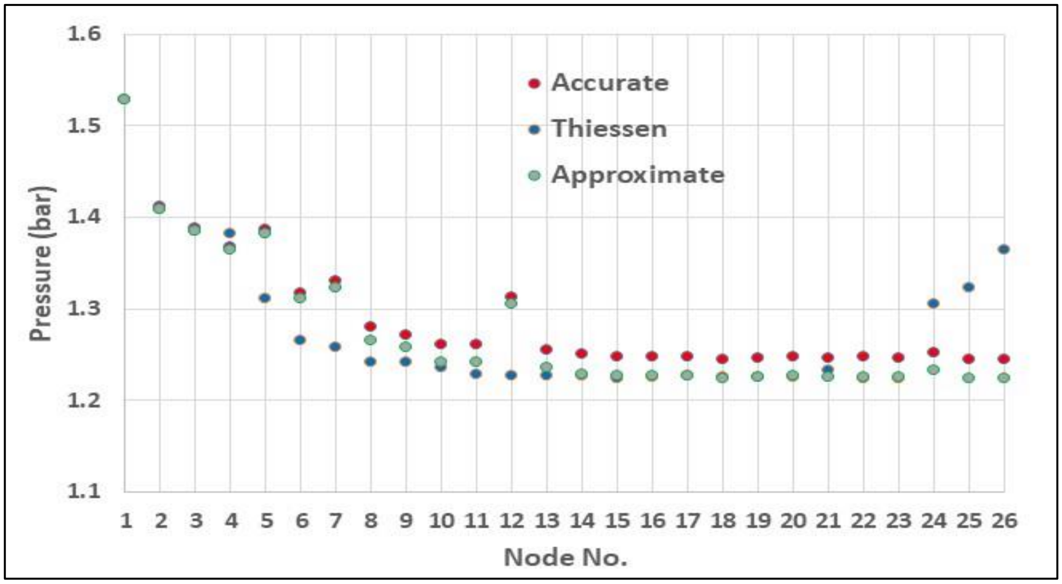

Figure 3: Nodal pressures among the three methods

Table 2 shows the results of resilience index measure for the approximate and Thiessen methods. Since in reality there are no actual demands at the intersection nodes, it is not possible to evaluate the resilience index for the accurate method. The Thiessen method showed higher value of the resilience index than the approximate method with a difference of about 4 ppm. This difference is related to the overestimation of nodal pressures at the end nodes of the network.

Table 2: Resilience indices for Thiessen and approximate methods

Method	Thiessen	Approximate
RI	0.11595881	0.11595488

7. Conclusion

A comparison study on the effect of network Skeletonization and water demand allocation methods was carried out in this paper. Two allocation methods were adopted, namely, the Thiessen and approximate methods. The hydraulic performance of each method is evaluated using the satisfaction of pressures at the network nodes considering all pipes in service, while measuring the hydraulic performance under pipe failure using the resilience index. The main result of the study is the overestimation of pressures at end nodes of the network when Thiessen method is used. This increase has resulted in an increase in the resilience index. The approximated method outperformed the Thiessen method at all network nodes. Due to its application simplicity and performance, the approximate method could be a preliminary choice for demand allocation. The study suggests extending the analysis for larger networks and using other hydraulic measures like network entropy.

References

1. Bhave, R.P., and Gupta, R., 2006. *Analysis of Water Distribution Networks*. UK: Alpha Science International Ltd.
2. Bentley WaterGEMS Connect Edition, Version 10.01.01.40, Product Data Sheet, www.bentley.com/en/resources/watergems [*accessed 6.12.2021*].
3. Jayaram, N., and Srinivasan, K., 2008. Performance-based optimal design and rehabilitation of water distribution networks using life cycle costing. Water Resources Research, 44(1).
4. Prasad, T.D., and Park, N.S., 2004. Multi-objective genetic algorithms for design of water distribution networks. Journal of Water Resources Planning and Management, 130(1), 73-82.
5. Tanyimboh, T.T., and Templeman, A.B., 1993a. Maximum entropy flows for single-source networks. Engineering Optimization, 22(1), 49-63.
6. Todini, E., 2000. Looped water distribution networks design using a resilience index-based approach. Urban Water, 2(2), 115-122.

7. Vaabel, J., Ainola, L., and Koppel, T., 2006. Hydraulic power analysis for determination of characteristics of a water distribution system. 8th Annual Water Distribution Systems Analysis Symposium, Cincinnati, Ohio, USA, 1-9.