Analysis of M/M/1/N Queueing System with Encouraged Arrivals, Feedback, Catastrophe and Retention of Reneged Customers Roman

***M. Akhdar[a], A. Alsileeni[b]**

[a]Department of Statistics and Econometrics, Faculty of Economics and Political Science, Tripoli University, Tripoli, Libya

[b]Collaborator, Department of Statistics, Faculty of Science, Al-Asmari University, Zliten ,Libya

*Crosspnding author: ***M.Akhder@uot.edu.ly***

Abstract

This paper aims to derive the analytical solution of the truncated single-channel queue: M/M/1/N at the steady state by adding the concepts of encouraging arrivals, feedback strategy, catastrophe, and retention of reneged customers. Using the iterative method obtain the probability that there are *n* customers in the system, the possibility of a vacuous system, and some performance effectiveness measures. Some important queueing models are exceptional cases of this system. Some numerical values are given for the additional concepts to show the effect of the (arrival rate, and catastrophe rate) on the model so that the companies implementing this model work to reduce the harmful ones.

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1. Introduction

In recent times, most of the major companies are striving to attract new customers and retain old ones. The discounts that companies make in certain seasons encourage customers to visit these companies, which encouraged companies to think about studying impatient customers to find solutions to the probability of their behavior. Several authors have investigated M/M/1/N queuing systems. To mention a few, in the year 2000, Sharma and Tarabia [1] have studied transient analysis of M/M/1/N queue without any concepts. In the year 2014, Kumar et al [2] have focused to study the Optimization of an M/M/1/N feedback queue with retention of reneged customers. Kalidass et al [3] discussed a time-dependent analysis of an M/M/1/N queue with catastrophes and a repairable server. Garg and Jain [4] have proposed a Distribution of the number of times the M/M/2/N queuing system reaches its capacity in time t under catastrophic effects. An M/M/1/N feedback queuing system with reverse balking, reverse reneging, and retention of reneged customers studied by Kumar and Som [5]. Som [6] study Cost-profit analysis of stochastic heterogeneous queue with reverse balking, feedback, and retention of impatient customers. In recent years, queuing systems with encouraged arrivals have been studied by Som and Seth [7, 8], Awasthi [9], Bouchentouf et al [10], Fazlollahtabar and Gholizadeh [11], Awasthi [12], Awasthi [13], Som [14]. Finally, Rao et al [15] Focused develop a finite capacity single server Markovian queueing system with encouraged or discouraged arrivals and a modified reneging policy of the customers.

In this paper, we proposed to derive the analytic solution for the steady-state M/M/1/N queuing system by adding the concepts of entrant encouragement, feedback, and return customer retention. The explicit probability of having *n* clients in the system, and the probability of getting an empty system using an iteration relationship. Some special cases and some measures of effectiveness are elicited. Finally, two concepts (arrival rate, and catastrophe rate) are simulated to see their effect on (the probability of no customers in the system, the expected number of customers in the system, average retention rate), holding the rest of the concepts.

2. Basic Notations and Assumptions

To construct the system of this paper, we define the following parameters:

$\lambda =$ Mean arrival rate.

$\mu =$ Mean service rate per service representative.

n = Number of customers in the system.

N= Capacity system.

$\rho = \lambda/\mu =$ Utilization factor.

$\upsilon =$ Catastrophe rate.

$R_r =$ The average reneging rate.

$R_R =$ The average retention rate.

$L_s =$ Expected number of customers in the system.

$L_q =$ Expected number of customers waiting to be served.

$p =$ The probability that a customer may leave the queue.

$1-p =$ The probability that a customer may remain in the queue for service.

$q =$ The probability that a customer joins the departure process.

$1-q =$ The probability that a customer joins the end of the original queue.

$\alpha =$ Reneging rate of a certain length of time which a customer will wait for service.

$\eta =$ Represents the percentage increase in the number of customers computed from past or observed data.

$p_n(t) =$ Transient state probability that there are exactly n customers in the system.

$p_0(t) =$ The probability that no customers are in the service department at time t.

$p_n =$ The steady-state probability that there are n customers in the system.

$p_0 =$ The steady-state probability that there are no customers in the system.

The assumptions of this model are listed as follows:

(i) Customers arrive at the server one by one according to the Poisson process with rate $\lambda(1+\eta)$.

(ii) Service time of the customers are independent and identically distributed (iid) exponential random variables with rate μ . And the customers are served according to FCFS discipline.

(iii) After completion of each service the customer either joins at the end of the original queue as a feedback customer with probability $(1-q)$ or departure the system with probability q .

(iv) After joining the queue each customer will wait a certain length of time for service, to begin with, probability $(1-p)$. If service has not begun by then, the customer will get impatient and leave the queue without getting service with a probability $(n-1)\alpha\, p, n \ge 2$.

(v) The catastrophe occurs at the service department according to the Poisson process with a rate υ when the system is not empty or empty. The occurrence of a catastrophe destroys all the customers in the instants and affects the system as well.

3. Model Formulation and Analysis

Applying Markovian conditions, we obtain the following probability differential-difference equations:

$$p_0'(t) = -[\lambda(1+\eta)+\upsilon]p_0(t) + q\mu\, p_1(t), \quad n = 0 \tag{1}$$

$$p_n'(t) = -[\lambda(1+\eta)+q\,\mu+(n-1)\alpha\, p+\upsilon]p_n(t) + (q\,\mu+n\alpha\, p)p_{n+1}(t) + \lambda(1+\eta)\, p_{n-1}(t), \quad 1 \le n \le N-1 \tag{2}$$

$$p_N'(t) = -[q\,\mu+(N-1)\alpha\, p+\upsilon]p_N(t) + \lambda(1+\eta)p_{N-1}(t), \quad n = N \tag{3}$$

4. Steady – State Solution

In the steady–state case it is known that $\lim_{t\to\infty} p_n'(t) = 0$ and, $\lim_{t\to\infty} p_n(t) = p_n$, the probability – difference equations are:

$$0 = -[\lambda(1+\eta)+\upsilon]p_0 + q\mu\, p_1, \quad n = 0 \tag{4}$$

$$0 = -[\lambda(1+\eta)+q\,\mu+(n-1)\alpha\, p+\upsilon]p_n + \lambda(1+\eta)\, p_{n-1} + (q\,\mu+n\alpha\, p)p_{n+1}, \quad 1 \le n \le N-1 \tag{5}$$

$$0 = -[q\,\mu+(N-1)\alpha\, p+\upsilon]p_N + \lambda(1+\eta)p_{N-1}, \quad n = N \tag{6}$$

Solving Equations (4) and (5), one has:

$$\begin{aligned}
&[q\,\mu+(n-1)\alpha\, p+\upsilon]p_n - \lambda(1+\eta)p_{n-1} \\
&= (q\,\mu+n\alpha\, p)p_{n+1} - \lambda(1+\eta)p_n \\
&= [q\,\mu+(n-1)\alpha\, p]p_n - \lambda(1+\eta)p_{n-1} \\
&= [q\,\mu+(n-2)\alpha\, p]p_{n-1} - \lambda(1+\eta)p_{n-2} \\
&= [q\,\mu+(n-3)\alpha\, p]p_{n-2} - \lambda(1+\eta)p_{n-3} \\
&= q\,\mu p_1 - \lambda(1+\eta)p_0 \\
&= \upsilon p_0
\end{aligned} \tag{7}$$

From Equation (7), we find:

$$p_n = \left[\frac{\lambda(1+\eta)p_{n-1} + \upsilon\, p_0}{q\mu+(n-1)\alpha\, p+\upsilon}\right], \quad 1 \le n \le N-1 \tag{8}$$

Replace *n-1* instead of *n* in Equation (8), to find:

$$p_n = \left\{\frac{[\lambda(1+\eta)]^2\, p_{n-2}}{[q\mu+(n-1)\alpha\, p+\upsilon][q\mu+(n-2)\alpha\, p+\upsilon]}\right\} + \left\{\frac{\lambda(1+\eta)}{[q\mu+(n-1)\alpha\, p+\upsilon][q\mu+(n-2)\alpha\, p+\upsilon]}\right.$$

$$+\frac{1}{q\mu+(n-1)\alpha\, p+\upsilon}\Bigg\}\upsilon\, p_0 \qquad (9)$$

Also, subrogate *n-2* instead of *n* in Equation (8), it get:

$$p_n=\Bigg\{\frac{[\lambda(1+\eta)]^3}{[q\mu+(n-1)\alpha\, p+\upsilon][q\mu+(n-2)\alpha\, p+\upsilon]}$$

$$\times\frac{1}{q\mu+(n-3)\alpha\, p+\upsilon}\Bigg\}p_{n-3}$$

$$+\Bigg\{\frac{[\lambda(1+\eta)]^2}{[q\mu+(n-1)\alpha\, p+\upsilon][q\mu+(n-2)\alpha\, p+\upsilon]}$$

$$\times\frac{1}{q\mu+(n-3)\alpha\, p+\upsilon}+\frac{\lambda(1+\eta)}{q\mu+(n-1)\alpha\, p+\upsilon}$$

$$\times\frac{1}{q\mu+(n-2)\alpha\, p+\upsilon}+\frac{1}{q\mu+(n-1)\alpha\, p+\upsilon}\Bigg\}\upsilon\, p_0 \qquad (10)$$

As well, replace *n-3* instead of *n* in Equation (8), we obtained:

$$p_n=\Bigg\{\frac{[\lambda(1+\eta)]^4}{[q\mu+(n-1)\alpha\, p+\upsilon][q\mu+(n-2)\alpha\, p+\upsilon]}$$

$$\times\frac{1}{q\mu+(n-3)\alpha\, p+\upsilon}\times\frac{1}{q\mu+(n-4)\alpha\, p+\upsilon}\Bigg\}p_{n-4}$$

$$+\Bigg\{\frac{[\lambda(1+\eta)]^3}{[q\mu+(n-1)\alpha\, p+\upsilon][q\mu+(n-2)\alpha\, p+\upsilon]}$$

$$\times\frac{1}{[q\mu+(n-3)\alpha\, p+\upsilon][q\mu+(n-4)\alpha\, p+\upsilon]}$$

$$+\frac{[\lambda(1+\eta)]^2}{[q\mu+(n-1)\alpha\, p+\upsilon][q\mu+(n-2)\alpha\, p+\upsilon]}$$

$$\times\frac{1}{q\mu+(n-3)\alpha\, p+\upsilon}+\frac{\lambda(1+\eta)}{q\mu+(n-1)\alpha\, p+\upsilon}$$

$$\times\frac{1}{q\mu+(n-2)\alpha\, p+\upsilon}+\frac{1}{q\mu+(n-1)\alpha\, p+\upsilon}\Bigg\}\upsilon\, p_0 \qquad (11)$$

Using Equations (4), (9), (10) and (11). The following general relation can be derived:

$$p_n = p_0 \left\{ \frac{[\lambda(1+\eta)]^n}{\prod_{k=1}^{n}[q\mu+(k-1)\alpha\, p+\upsilon]} + \upsilon \sum_{j=0}^{n-1} \frac{[\lambda(1+\eta)]^{n-(j+1)}}{\prod_{i=j}^{n-1}(q\mu+i\,\alpha\, p+\upsilon)} \right\}, \qquad 1 \le n \le N-1 \qquad (12)$$

Finally, from Equation (6) and relation (8) with $n=N$, one has:

$$p_N = p_0 \left\{ \frac{[\lambda(1+\eta)]^N}{\prod_{k=1}^{N}[q\mu+(k-1)\alpha\, p+\upsilon]} + \upsilon \sum_{j=0}^{N-1} \frac{[\lambda(1+\eta)]^{N-(j+1)}}{\prod_{i=j}^{N-1}(q\mu+i\,\alpha\, p+\upsilon)} \right\}, \qquad n = N \qquad (13)$$

From Equations (12) and (13), one has:

$$p_n = p_0 \left\{ \frac{[\lambda(1+\eta)]^n}{\prod_{k=1}^{n}[q\mu+(k-1)\alpha\, p+\upsilon]} + \upsilon \sum_{j=0}^{n-1} \frac{[\lambda(1+\eta)]^{n-(j+1)}}{\prod_{i=j}^{n-1}(q\mu+i\,\alpha\, p+\upsilon)} \right\}, \qquad 1 \le n \le N \qquad (14)$$

From Equations (4) and (14), and using the normalizing condition $p_0 + \sum_{n=1}^{N} p_n = 1$ on has:

$$p_0^{-1} = 1 + \sum_{n=1}^{N} \left\{ \frac{[\lambda(1+\eta)]^n}{\prod_{k=1}^{n}[q\mu+(k-1)\alpha\, p+\upsilon]} + \upsilon \sum_{j=0}^{n-1} \frac{[\lambda(1+\eta)]^{n-(j+1)}}{\prod_{i=j}^{n-1}(q\mu+i\,\alpha\, p+\upsilon)} \right\} \qquad (15)$$

5. Measures of effectiveness

In this paper, we will work on expected the number of customers in the system and the queue, as well the average rates of reneging and retention of reneged customers, as follows:

(i) The expected number of customers in the system is given by:

$$L_s == \sum_{n=0}^{N} np_n = \sum_{n=1}^{N} n \left\{ \frac{[\lambda(1+\eta)]^n}{\prod_{k=1}^{n} [q\mu + (k-1)\alpha\, p + \upsilon]} \right.$$

$$\left. + \upsilon \sum_{j=0}^{n-1} \frac{[\lambda(1+\eta)]^{n-(j+1)}}{\prod_{i=j}^{n-1} (q\mu + i\,\alpha\, p + \upsilon)} \right\} p_0, \tag{16}$$

(ii) The expected number of customers in the queue is given by:

$$L_q = L_s - (1 - p_0), \tag{17}$$

(iii) The average reneging rate is given by:

$$R_r = \sum_{n=1}^{\infty} (n-1)\alpha\, p\, p_n, \tag{18}$$

(iv) The average retention rate is given by:

$$R_R = \sum_{n=1}^{\infty} (n-1)\alpha(1-p)\, p_n \tag{19}$$

where p_n and p_0 are given in relations (14) and (15) respectively.

6. Special Cases

Some queuing systems can be gotten as special cases of this system:

Case (1): Consider $\upsilon = 0$, this is the queueing system M/M/1/N with encouraged arrivals, feedback, and retention of reneged customers. Then relations (14), (15), and (16) are expressed as:

$$p_n = p_0 \left\{ \frac{[\lambda(1+\eta)]^n}{\prod_{k=1}^{n} [q\mu + (k-1)\alpha\, p]} \right\}, \quad 1 \le n \le N, \tag{20}$$

$$L_s = \sum_{n=1}^{N} n \left\{ \frac{[\lambda(1+\eta)]^n}{\prod_{k=1}^{n} [q\mu + (k-1)\alpha\, p]} \right\} \tag{21}$$

wither

$$p_0^{-1} = 1 + \sum_{n=1}^{N} \left\{ \frac{[\lambda(1+\eta)]^n}{\prod_{k=1}^{n} [q\mu + (k-1)\alpha\, p]} \right\} \tag{22}$$

Relations (20)- (22) are the same results Som [14].

Case (2): Consider $\upsilon = 0$ and, $q = 1$, this is the queueing system M/M/1/N with encouraged arrivals and retention of reneged customers. Then relations (14), (15), and (16) are expressed as:

$$p_n = p_0 \left\{ \frac{[\lambda(1+\eta)]^n}{\prod_{k=1}^{n} [\mu + (k-1)\alpha\, p]} \right\}, \quad 1 \le n \le N, \tag{23}$$

$$L_s = \sum_{n=1}^{N} n \left\{ \frac{[\lambda(1+\eta)]^n}{\prod_{k=1}^{n} [\mu + (k-1)\alpha\, p]} \right\} \tag{24}$$

wither

$$p_0^{-1} = 1 + \sum_{n=1}^{N} \left\{ \frac{[\lambda(1+\eta)]^n}{\prod_{k=1}^{n} [\mu + (k-1)\alpha\, p]} \right\} \tag{25}$$

Relations (23)-(25) are the same results Kumar [16].

Case (3): Let $\alpha = 0, \upsilon = 0, q = 1$ and $p = 1$, this is the queueing system M/M/1/N encouraged arrivals Then relations (14), (15), and (16) are expressed as:

$$p_n = p_0 \left[\frac{\lambda(1+\eta)}{\mu} \right]^n, \quad 1 \le n \le N, \tag{26}$$

$$L_s = \sum_{n=1}^{N} n \left[\frac{\lambda(1+\eta)}{\mu} \right]^n \tag{27}$$

wither

$$p_0^{-1} = 1 + \sum_{n=1}^{N} \left[\frac{\lambda(1+\eta)}{\mu} \right]^n \tag{28}$$

Relations (26)-(28) are the same results Som [17].

Case (4): Let $\alpha = 0, \upsilon = 0, q = 1, \eta = 0$ and $p = 1$, this is the queueing system M/M/1/N without any concepts. Then relations (14), (15), and (16) are expressed as:

$$p_n = \frac{1-(\lambda/\mu)}{1-(\lambda/\mu)^{N+1}}(\lambda/\mu)^n, \quad 0 \le n \le N, \tag{29}$$

$$L_s = \sum_{n=1}^{N} n\left\{\frac{1-(\lambda/\mu)}{1-(\lambda/\mu)^{N+1}}(\lambda/\mu)^n\right\} \tag{30}$$

Relations (29) and (30) are the same results Boxma [18].

Case (5): Let $\alpha = 0, \upsilon = 0, q = 1, \eta = 0, p = 1$ and $N \to \infty$, this is the queueing system M/M/1 with without any concepts Then relations (14), (15) and (16) are expressed as:

$$p_n = [1-(\lambda/\mu)](\lambda/\mu)^n, \quad n \ge 0, \tag{31}$$

$$L_s = (1-(\lambda/\mu))\sum_{n=1}^{\infty} n\,(\lambda/\mu)^n = \frac{\lambda/\mu}{1-\lambda/\mu} \tag{32}$$

Relations (31) and (32) are the same results [19].

7. An Illustrative Example

In this paper, we randomly assign values for the parameters $\lambda, \mu, \upsilon, \alpha, q, p$, and η to see the effect of their movement on the queue M/M/1/N. And that is as follows:

Assume that $q = 0.3, p = 0.4, \eta = 0.5, \mu = 8$ $, \alpha = 0.25, N = 10$ and $\upsilon = 0.9$, then substituting these values into equations (15), (16), and (19) respectively was entered for the Mat CAD program, one obtain the results shown in Table 1:

Table 1: The computed values of the M / M / 1/N queue characteristics with changing the values of the parameter λ.

λ	p_0	L_s	R_R
2.00	0.07400	4.901	0.596
2.50	0.03700	5.852	0.733
3.00	0.01500	6.873	0.883
3.25	0.00943	7.313	0.948
3.75	0.00355	7.995	1.050
4.40	0.00103	8.556	1.134
4.90	0.00042	8.830	1.175
5.30	0.00021	8.988	1.198
5.70	0.00011	9.111	1.217
6.50	0.00003	9.287	1.243

For further clarification, the contents of Table 1 have been graphically represented as follows:

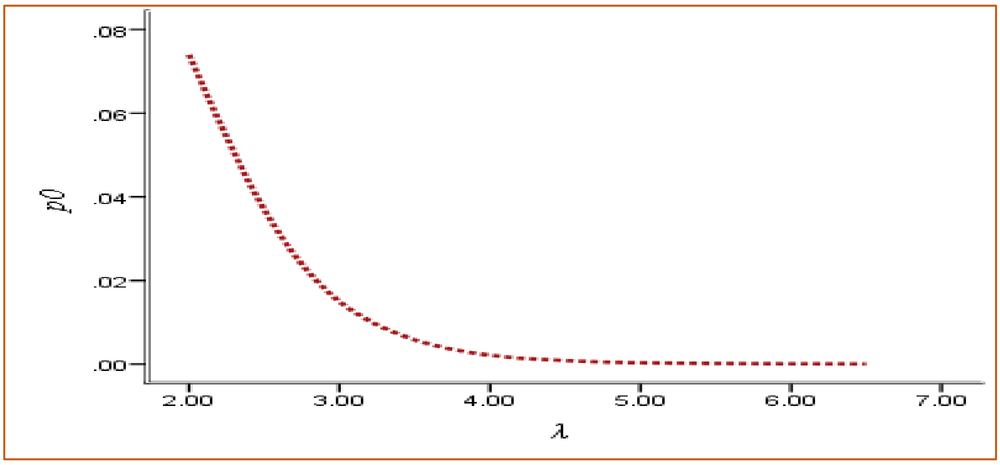

Figure 1: the relation p_0 & λ

From Table 1 and Figure 1, we can see that the increase in arrival rate leads to a decrease in the probability of no customers in the system. That is, the results are consistent with the functioning of the model.

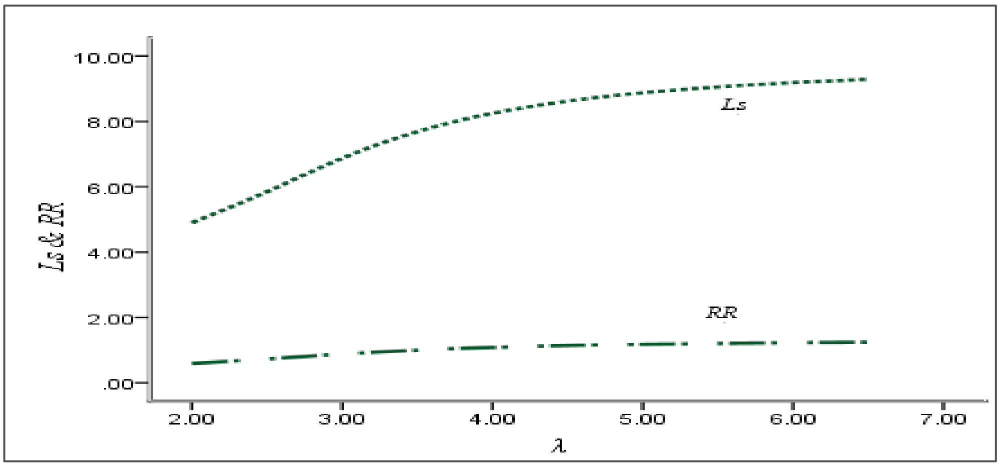

Figure 2: the relation (L_s, R_R) & λ

From Table 1, Figure 2, it is visible that the increase in the average arrival rate increases the expected system size. On the other hand, the rate of average retention increases gradually due to increasing system size that leads to a high level of high confidence of customers with a particular firm. That is, the increase in the arrival rate positively affects the system.

Also, assume that $q = 0.3, p = 0.4, \eta = 0.5$ $, \mu = 8, \alpha = 0.25, N = 10$ and $\lambda = 3.75$, one obtain the results shown in Table 2:

Table 2: The computed values of the M / M / 1/N queue characteristics with changing the values of the parameter υ

υ	p_0	L_s	R_R
0.1	0.00061	8.723	1.159
0.3	0.00102	8.562	1.134
0.4	0.00129	8.475	1.121

0.7	0.00245	8.196	1.080
0.9	0.00355	7.995	1.050
1.0	0.00421	7.891	1.034
1.5	0.00861	7.358	0.955
1.7	0.01100	7.149	0.924
2.1	0.01600	6.761	0.866
2.2	0.01700	6.671	0.853

For further clarification, the contents of Table 2 have been graphically represented as follows:

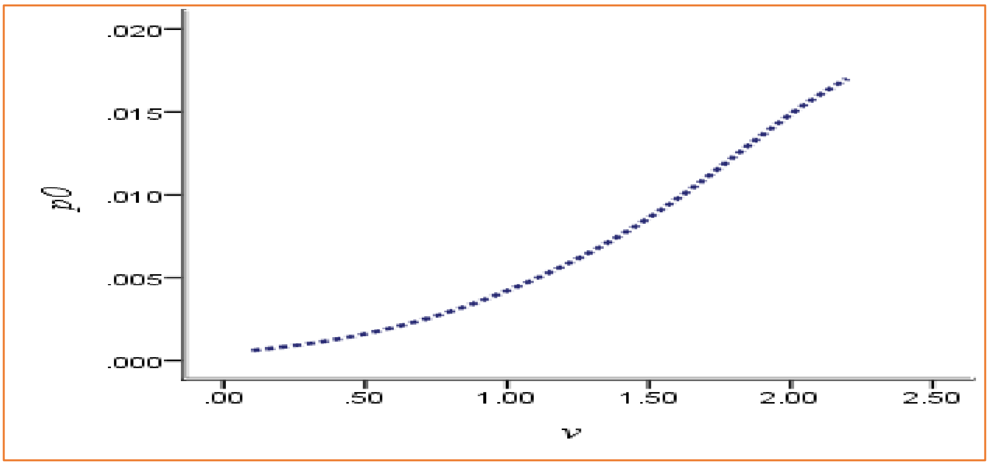

Figure 3: the relation p_0 & υ

From Table 2 and Figure 3, we can see that the increase in catastrophe rate leads to an increase in the probability of no customers in the system. That is, the results are consistent with the functioning of the model.

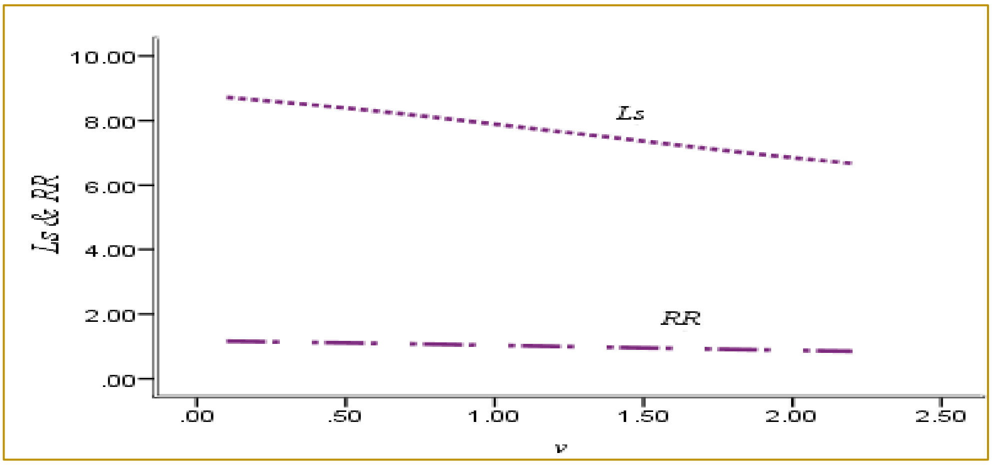

Figure 4: the relation (L_s, R_R) & υ

From Table 2, and Figure 4, is visible that the increase in the catastrophe rate decreases the expected system size accompanied by a slow decrease in the average retention rate. That is, the increase in the catastrophe rate negatively affects the system. Therefore, the concerned companies should try to reduce the catastrophe rate to the largest possible extent in order to be able to attract customers.

8. Conclusion and recommendations

This paper extracted the steady-state solution of the M / M / 1/N queue with encouraging arrivals, feedback, catastrophe, and retention of reneged customers. The probabilities of having *n* customers in the system, some performance measures, and as well as some queues which are special cases were obtained. The catastrophe rate harms the system whereas the arrival rate has a positive effect. Meaning the more a company can reducing the catastrophe rate, the more customers trust it. For future work, this queue could be studied in the case of transient behavior with the addition of repair.

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