A Novel Denoising Method Based on Discrete Linear Chirp Transform

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ABSTRACT

Denoising of chirp based signals is a challenging problem in signal processing and communications. In this paper, we propose a suitable denoising algorithm based on the discrete linear chirp transform (DLCT), which provides local signal decomposition in terms of linear chirps. Analytical expression for the optimal filter response is derived. The method relies on the ability of the DLCT for providing a sparse representation to a wide class of broadband signals like chirp signals. Simulation results show the efficiency of the proposed method, especially for mono-component chirp signals.

Keywords: Discrete linear chirp transform, denoising, chirp signals, nonstationary signal filtering.

1 Introduction

Chirp signals, known as linear frequency modulated signals, are important class of nonstationary signals which frequently encountered in many practical applications, such as radar, sonar, and telecommunications. Noise is the main factor which influences the transmission and recognition of signals in communications, radar, sonar and optics. In order to reduce the influences caused by noise, several methods and algorithms have been proposed for denoising. Unfortunately, most of these algorithms were not efficient for nonstationary signals, such as chirp signals, which are characterized with time–frequency identities. In general methods or algorithms that are based on frequency domain only are not effective for such type of signals [1].

To deal with nonstationary signals, several algorithms are presented in the literature of denoising chirp based signals. Most of them are based on extending Fourier representations capable of providing instantaneous–frequency information for multi–component signals. These algorithms can be achieved by considering polynomial–phase transform [2], or second–order polynomial transforms [3-4]; however, the latter is preferable due to computational viability. Furthermore, a parametric characterization of the instantaneous frequency of each of the components [5] provides a realistic view of the evolving nature of the signal. Although procedures based on the chirplet transform [6–7], and polynomial chirplet transform (PCT) [8] have been proposed, their numerical implementation is difficult to obtain because of no straightforward way to solve non–convex optimization problems with multiple extrema.

In one hand, the fractional Fourier transform (FrFT) proposed by Namias [9] in 1980 has drawn a considerable amount of attention in analysing and processing of nonstationary chirp based signals. It is a generalization of the conventional Fourier transform. It has been applied to different problems in signal processing including signal separation and filtering [10–15]. The FrFT provides a continuous representation of a signal from the time to the frequency domain at intermediate domains by means of the fractional order of the transform. On the other hand, the DLCT is also an extension of the discrete Fourier transform (DFT) and provides a parametric modelling of the instantaneous frequencies of the components. It is introduced to represent a signal as a combination of linear chirps [4]. The DLCT is implemented efficiently using the fast Fourier transform (FFT) algorithm and can be applied for compression of nonstationary signals [16]. Unlike the FrFT in doing joint time–frequency representation, the DLCT is a joint chirp–rate frequency transformation, which can do a better job in denoising chirp based (nonstationary) signals. It has been shown in [17] that the discrete linear chirp transform has better performance than the fractional Fourier transform in terms of sparsity, computation time, and peak location.

In [18], a DLCT denoising algorithm which is used to obtain an estimate for the desired signal is proposed. The algorithm relies on the ability of the DLCT to decompose a signal iteratively into its components locally. Each of these components is filtered separately and then synthesized with the other filtered components to give the filtered signal. Since each segment of the signal has different components with different bandwidths, the filter has to be time–varying filter. The results show that the DLCT algorithm provides better performance than the FrFT algorithm.

In this paper a novel denoising method based on the discrete linear chirp transform is proposed. The designed filter approach uses the minimization criterion between the desired signal and the filtered signal to obtain optimal filter coefficients. The presented method shows an improvement in the performance compared with the conventional DLCT algorithm given in [18] for mono–component chirp signals.

The rest of the article is organized as follows. Section II presents the discrete linear chirp transform. In Section III, the proposed filter design technique is introduced. Simulation results are given in Section IV. Conclusions and future work are shown in Section V.

2 Discrete Linear Chirp Transform

Consider a discrete time chirp based analytic signal $x(n)$, where $n = 0,1, \dots, N-1$ and N is the number of samples embedded in a complex white Gaussian noise $\eta(0,\sigma^2)$. Then the observation signal $y(n)$ can be modeled as

$$y(n) = x(n) + \eta(n) \tag{1}$$

The signal $x(n)$ can be well estimated using the DLCT [2], which its pairs are given as follows

$$X(k,m) = \sum_{n=0}^{N-1} x(n)\, exp\left(-j\frac{2\pi}{N}(C\, m\, n^2 + k\, n)\right) \tag{2}$$

$$x(n) = \frac{1}{N} \sum_{m=-L/2}^{L/2-1} \sum_{k=0}^{N-1} X(k,m)\, exp\left(j\frac{2\pi}{N}(C\, m\, n^2 + k\, n)\right) \tag{3}$$

where C is the resolution of the transform in the chirp-rate domain and L is an even integer number as defined in [2]. The DLCT attempts to decompose a signal using discrete linear chirps characterized by a discrete frequency $2\pi k/N$, and a chirp–rate $\beta = C\, m$.

For certain chirp-rate domain, we can define the DLCT pairs in the matrix form as

$$\boldsymbol{X} = \boldsymbol{D}_{\beta}\, \boldsymbol{x} \quad \text{and} \quad \boldsymbol{x} = \boldsymbol{D}_{-\beta}\, \boldsymbol{X} \tag{4}$$

where $\boldsymbol{X}, \boldsymbol{x}$ are $N \times 1$ vectors and $\boldsymbol{D}_{\beta}, \boldsymbol{D}_{-\beta}$ are $N \times N$ matrices given as

$$\boldsymbol{D}_{\beta}(k,n) = exp\left(-j\frac{2\pi}{N}(k\, n + \beta\, n^2)\right) \tag{5}$$

and

$$\boldsymbol{D}_{-\beta}(k,n) = exp\left(j\frac{2\pi}{N}(k\, n + \beta\, n^2)\right) \tag{6}$$

Since the DLCT is a unitary transformation, the DLCT matrices in (5) and (6) are related by $\boldsymbol{D}_{\beta} = \boldsymbol{D}^{H}_{-\beta}$, where $(.)^H$ denotes the conjugate transpose operation, $\boldsymbol{D}_{\beta}\, \boldsymbol{D}^{H}_{-\beta} = \boldsymbol{I}$, and $\boldsymbol{I}$ is the identity matrix.

3 Proposed Filter Design Technique Using the DLCT

It is well known that the optimal filter design for stationary process time–invariant signals can be achieved with Wiener filter which can be implemented efficiently using fast Fourier transform. However, for time–varying nonstationary signals such as chirp signals, we need a transform that can handle these time variations in a more efficient way. Therefore, the DLCT will be used to represent such signals. In this paper, we will adopt a similar criteria used in Wiener filter design which is the minimization of the average square error.

Our design problem requires that we find the filter response, $\boldsymbol{g}$, that minimizes the average square error over M realizations as follows

$$\boldsymbol{g} = \arg \min_{\boldsymbol{g}} \frac{1}{M} \sum_{i=1}^{M} \|\widehat{\boldsymbol{x}}_i - \boldsymbol{x}\|^2 \tag{7}$$

where $\widehat{\boldsymbol{x}}_i$ is the DLCT inverse for the multiplication of the observation signal $\boldsymbol{y}_i$ with the filter response $\boldsymbol{g}$ as

$$\hat{\boldsymbol{x}}_i = \boldsymbol{D}_{-\beta}\boldsymbol{G}\,\boldsymbol{D}_{\beta}\boldsymbol{y}_i \tag{8}$$

and $\boldsymbol{G}$ is an $N \times N$ diagonal matrix whose elements are $\boldsymbol{g}$ =diag$(\boldsymbol{G}) = [g_0, g_1, \dots, g_{N-1}]$. If we let $\boldsymbol{s}_i = \boldsymbol{D}_\beta\ \boldsymbol{y}_i$ and since $\boldsymbol{G}$ is a diagonal matrix, then we can write $\boldsymbol{G}\ \boldsymbol{s}_i = \boldsymbol{S}_i\ \boldsymbol{g}$, where $\boldsymbol{S}_i$ is an $N \times N$ diagonal matrix defines as $\boldsymbol{s}_i$ =diag$(\boldsymbol{S}_i)$, given that $\overline{\boldsymbol{S}}_i = \boldsymbol{D}_{-\beta}\boldsymbol{S}_i$. Thus, we can rewrite our quadratic cost function $J(\boldsymbol{g})$ such as

$$J(\boldsymbol{g}) = \frac{1}{M}\sum_{i=1}^{M}(\overline{\boldsymbol{S}}_i\boldsymbol{g} - \boldsymbol{x})^H\ (\overline{\boldsymbol{S}}_i\boldsymbol{g} - \boldsymbol{x}) \tag{9}$$

or equivalently,

$$J(\boldsymbol{g}) = \frac{1}{M}\sum_{i=1}^{M}\left(\boldsymbol{g}^H\overline{\boldsymbol{S}}_i^H\overline{\boldsymbol{S}}_i\boldsymbol{g} - 2\mathcal{R}(\boldsymbol{x}^H\overline{\boldsymbol{S}}_i\boldsymbol{g}) + \boldsymbol{x}^H\boldsymbol{x}\right) \tag{10}$$

where $\mathcal{R}(.)$ is the real part. Equation (10) can be expressed in the following form

$$J(\boldsymbol{g}) = \boldsymbol{g}^H\boldsymbol{Q}\boldsymbol{g} - 2\boldsymbol{b}^T\boldsymbol{g} + a \tag{11}$$

where,

$$\boldsymbol{Q} = \frac{1}{M}\sum_{i=1}^{M}\overline{\boldsymbol{S}}_i^H\overline{\boldsymbol{S}}_i\ ,$$

$$\boldsymbol{b} = \frac{1}{M}\sum_{i=1}^{M}[2\mathcal{R}(\boldsymbol{x}^H\overline{\boldsymbol{S}}_i)]^T\ \ , and$$

$$a = M\ \|\boldsymbol{x}\|^2$$

The optimal response of the filter $\boldsymbol{g}_{opt}$ is the solution of minimization the cost function $J(\boldsymbol{g})$ and this can be done by computing the gradient of (11) with respect to $\boldsymbol{g}$, which yields

$$\boldsymbol{g}_{opt} = \boldsymbol{Q}^{-1}\boldsymbol{b} \tag{12}$$

Since $\boldsymbol{Q}$ is a diagonal matrix, its inverse always exists. So far we consider only the minimization of the average square error with respect to the filter response $\boldsymbol{g}$. Now, how do we choose the other parameter in the cost function which is the chirp–rate β ? Trying to find the optimal β analytically is not an easy problem to solve. In this paper, we use an approach based on the DLCT. We compute the DLCT of the observation signal and find the chirp–rate that maximizes it.

In (12), the optimal solution $\boldsymbol{g}_{opt}$ depends on the desired signal $\boldsymbol{x}$. An estimate for the desired signal can be obtained using the algorithm given in [18].

4 Results and Discussion

To demonstrate the performance of the proposed DLCT denoising algorithm, two examples are performed using a synthetic signal and a real–world signal. In each example, we compare the mean square error (MSE) of the proposed method with the DLCT filtering algorithm given in [18]. The estimated signal using the DLCT algorithm is used as the desired signal for the proposed method with a set of 20 realizations.

For the case of synthetic signal, we use the signal given in Figure 1(a). Figure 1(b) and (c) show the noisy chirp signal, and the denoised chirp signal using optimal DLCT filtering method, respectively. In Figure 1(d), we present the MSE for various algorithms against signal–to–

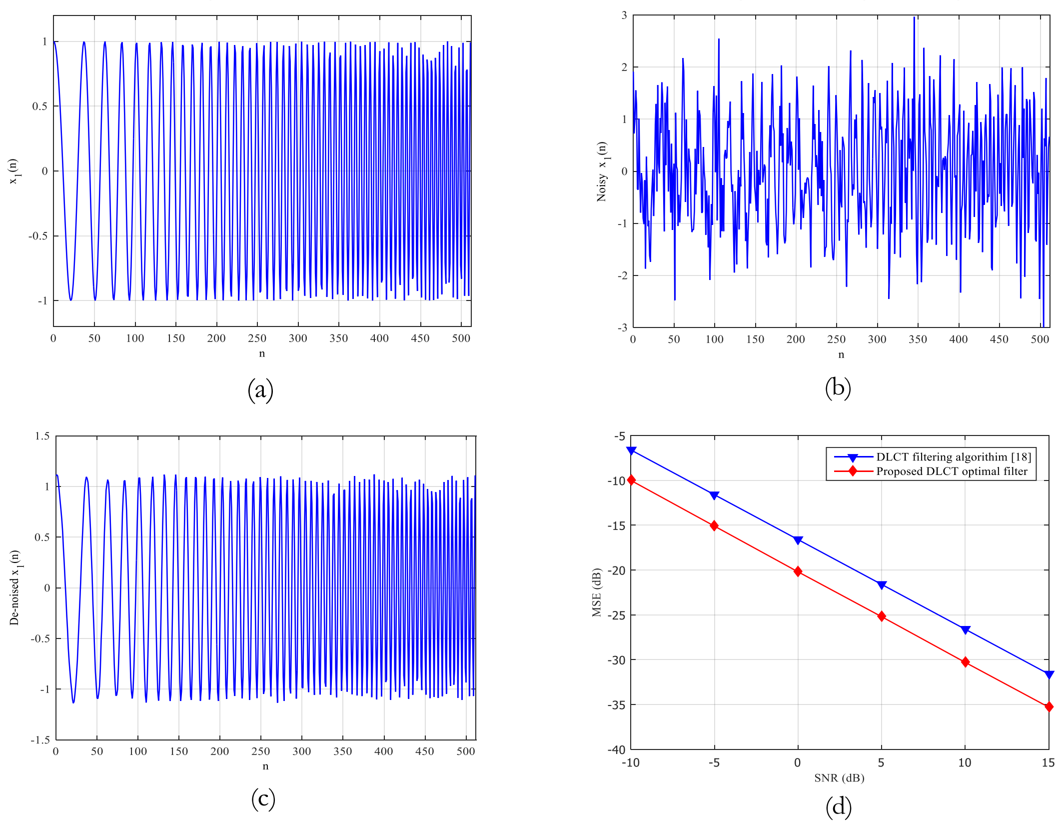

Figure 1: *Linear chirp signal: (a) the chirp signal, (b) the noisy chirp signal with SNR=0 dB, (c) the filtered chirp signal using the optimal DLCT filtering method, (d) the mean square error for the DLCT and optimal DLCT methods.*

noise ratio (SNR). It can be seen that the proposed optimal filtering algorithm outperforms the performance of the DLCT filtering algorithm.

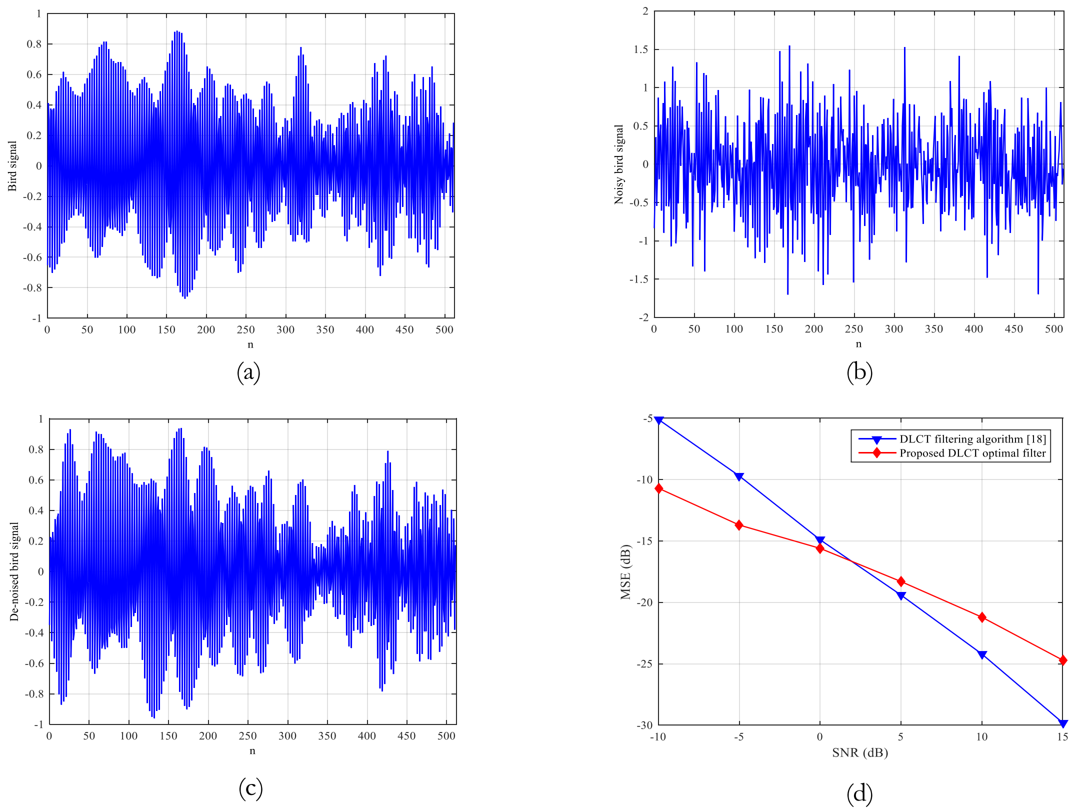

Figure 2: *Real–world signal: (a) the bird chirping signal, (b) the noisy bird chirping signal with SNR=0 dB, (c) the filtered bird chirping signal using optimal DLCT filtering method, (d) the MSE for various methods.*

To quantify the MSE improvement, a real–world signal (bird chirping signal) with varying noise level is also simulated. The noiseless and corresponding corrupted bird chirping signals are presented in Figure 2(a) and (b). The denoised bird chirping signal based on the optimal DLCT filtering algorithm is shown in Figure 2(c) at SNR=0 dB. Similar to the previous example, Figure 2(d) depicts the MSE for the proposed method compared with the conventional DLCT filtering algorithm [18] as a function of the input SNR, where input SNR is varied from -10 dB (severely poor SNR) to 15 dB (high SNR). The DLCT filtering algorithm greatly enhances the MSE for low SNR levels. It is shown that for proposed filtering method, the performance is not optimal since it achieves worse MSE at high SNR. This is because the signal has many components with different chirp rates.

5 Conclusions

In this paper, an optimal filtering method based on the discrete linear chirp transform is proposed. The design of the filter is carried out and a closed form solution for the impulse response of the filter is derived. The performance of the proposed filter is compared with other algorithms. Simulation results show that the DLCT optimal filtering method outperform the performance of the conventional DLCT filtering algorithm for mono–component chirp signals. However, for multi–component chirp signals, the proposed filter gives better performance in low signal–to–noise ratio environment. As a future work, we will look for another approach that can deal with multicomponent signals contaminated in noise more optimally.

References

[1] O. A. Alkishriwo, L. F. Chaparro, and A. Akan, "Signal separation in the Wigner distribution using fractional Fourier transform," European Signal Processing Conf., EUSIPCO, Spain, September 2011, pp. 1879-1883.

[2] S. Peleg and B. Friedlander, "The discrete polynomial-phase transform," IEEE Transactions on Signal Processing, vol. 42, no. 8, pp. 1901-1914, August 1995.

[3] M. Z. Ikram, K. Abed-Meraim, and Y. Hua, "Fast quadratic phase transform for estimating the parameters of multicomponent chirp signals," Digital Signal Processing, vol. 7, no. 2, pp. 127-135, 1997.

[4] O. A. Alkishriwo and L. F. Chaparro, "A Discrete Linear Chirp Transform (DLCT) for Data Compression," in Proc. of the IEEE International Conf. on Information Science, Signal Processing and their Applications, Montreal, Canada, July 2012, pp. 1283-1288.

[5] Y. Li, H. Fu, and P. Y. Kam, "Improved, approximate, time–domain ML estimators of chirp signal parameters and their performance analysis," IEEE Transactions on Signal Processing, vol. 57, no. 4, pp. 1260-1272, April 2009.

[6] Q. Yin, S. Qian, and A. Fing, "A fast refinement for adaptive Gaussian chirplet decomposition," IEEE Transactions on Signal Processing, vol. 50, no. 6, pp. 1298-1306, June 2002.

[7] Y. Lu, R. Demirli, G. Cardoso, and J. Saniie, "A successive parameter estimation algorithm for chirplet signal decomposition," IEEE Transactions on Signal Processing, vol. 53, no. 11, pp. 2121-2131, November 2006.

[8] Z. K. Peng, G. Meng, F. L. Chu, Z. Q. Lang, W. M. Zhang, and Y. Yang, "Polynomial chirplet transform with application to instantaneous frequency estimation," IEEE Transactions on Instrumentation and Measurement, vol. 60, no. 9, pp. 3222-3229, September 2011.

[9] V. Namias, "The fractional order Fourier transform and its application to quantum mechanics," IMA Journal of Applied Mathematics, vol. 25, , no. 3, pp. 241-265, 1980.

[10] L. B. Almeida, "The fractional Fourier transform and time–frequency representations," IEEE Transactions on Signal Processing, vol. 42, no. 11, pp. 3084-3091, November 1994.

[11] H. M. Ozaktas, Z. Zalevsky, and M. A. Kutay, The Fractional Fourier transform with applications in optics and signal processing, John Wiley, New York, USA, 2001.

[12] E. Sejdic, I. Djurovic, and L. Stankovic, "Fractional Fourier transform as a signal processing tool: an overview of recent developments," Signal Processing, vol. 91, no. 6, pp. 1351-1369, 2011.

[13] L. Durak and S. Aldirmaz, "Adaptive fractional Fourier domain filtering," Signal Processing, vol. 90, no. 4, pp. 1188-1196, April 2010.

[14] K. Prajna and C. K. Mukhopadhyay, "Fractional Fourier transform based adaptive filtering techniques for acoustic emission signal enhancement," Journal of Nondestructive Evaluation, vol. 14, pp. 1-15, January 2020.

[15] L. Wu, Y. Zhao, L. He, S. He, and G. Ren, "A time–varying filtering algorithm based on short–time fractional Fourier transform," International Conference on Computing, Networking, and Communications (ICNC), Big Island, Hi, USA, February 2020, pp. 555–560.

[16] O. A. Alkishriwo and L. F. Chaparro, "Signal compression using the discrete linear chirp transform (DLCT)," in Proc. of IEEE 20th European Signal Processing Conference (EUSIPCO 2012), Bucharest, Romania, Aug. 2012, pp. 2128-2132.

[17] O. A. Alkishriwo, "The discrete linear chirp transform and its applications," PhD Thesis, University of Pittsburgh, 2013.

[18] O. A. Alkishriwo, A.A. Elghariani, A. Akan, "Iterative time–varying filter algorithm based on discrete linear chirp transform," First Conference for Engineering Sciences and Technology (CEST-2018), Garaboulli, Libya, pp. 86-92, September 2018.