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Simulation–Based Comparison of Estimated Methods for the Differencing Fractional Parameter in ARFIMA Model

Mustafa A. Almahmoudi[1], Rida M. Khaga[2], and Maryouma E. Enaami[3]
Department Of Statistics, Faculty of Science, Tripoli University, Libya
[1]Mustafa.almahmoudi@gmail.com; [2]r.khaga@uot.edu.ly; [3]Ma.enaami@uot.edu.ly

Abstract

Long-range dependence (LRD) is a statistical measure for the speed of the autocorrelation function's decay for a time series. A process that is said to have long memory if its autocorrelation function decays is hyperbolic, instead of an exponential rate as the lag increase. Some of the time series data persists towards non-stationary in the long run data. The effort of differencing seems to be good solution towards the non-stationary counter parts. With regard to the above matter, this research presents the usefulness of autoregressive fractionally integrated moving average ARFIMA model as the solution towards the non-stationary persistency of time series in the long run data. In this paper, we analyze the estimation of the degree of differencing d in ARFIMA (0, d, 0) process, when the d belongs to the interval (0, 0.5). We present a simulation study for the estimators of d by using periodoagram $\widehat{\mathbf{d_p}}$, smoothed period-diagram $\widehat{\mathbf{d_{sp}}}$ and Whittle $\widehat{\mathbf{d_w}}$ methods with different sample sizes (64, 128, 256, 512, 1024, 2048) and 1000 repetitions for each sample. In general, as sample size (n) increases the estimators get even better, except for the $\widehat{\mathbf{d_{sp}}}$ estimator. Furthermore, the Whittle estimator $\widehat{\mathbf{d_w}}$ seems to be more accurate than the other estimators. The testing of hypothesis results showed that, the estimates of the fractional differencing parameter d by Whittle's method has good performance as increasing of sample sizes and d. Also based on the autocorrelation function, the results observed exhibitvery slow correlation decay which means that the process has a long memory.

KEY WORDS: Time Series; ARIMA; ARFIMA; Long-range dependence (LRD).

المستخلص

الاعتماد طويل المدى هو مقياس إحصائي يقيس سرعة اضمحلال دالة الارتباط الذاتي في السلاسل الزمنية. ويقال أن السلسلة الزمنية لها خاصية الذاكرة الطويلة إذا كانت دالة الارتباط الذاتي تضمحل ببطء ولا تأخذ شكل الدالة الآسية كما في السلاسل الزمنية بعض من بيانات السلاسل الزمنية تحافظ على عدم استقرارها على المدى الطويل، لذلك فإن ذات المدى القصير (الذاكرة القصيرة).

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إجراء عملية التفريق على السلاسل الزمنية يعتبر حلاً لتحويلها إلى سلاسل مستقرة. فيما يتعلق بهذا الأمر، يقدم هذا البحث أهمية استخدام الانحدار الذاتي الكسرى التكاملي المتوسط المتحرك ARFIMA كحل لأستقرر بيانات المدى الطويل للسلاسل الزمنية. في هذه الورقه نحلل قيمة معامل التفريق الكسرى (d) المقدرة في النموذج ذو الرتبة (0, d, 0) عندما ينتمي هذا المعامل للفترة (0 , 0.5). باستخدام دراسة المحاكاة تم تقدير قيمة (d) بثلاثة طرق هي: (Periodoagram Estimator $\widehat{d_p}$ –Smoothed Periodoagram Estimator $\widehat{d_{sp}}$ – Whittle $\widehat{d_w}$ Estimator) بأحجام عينات مختلفة (2048،1024،512،256،128،64) مع تكرار العملية 1000مره لكل عينة. بصفة عامة، بزيادة حجم العينة يصبح المقدرات افضل، باستناء المقدر $\widehat{d_{sp}}$, علاوة على دلك, مقدر $\widehat{d_w}$ اكثر دقة من المقدرين الاخرين. وأظهرت نتائج اختبار الفروض أن تقديرات المعلمة d بواسطة طريقة Whittle لها أداء جيد عند زيادة أحجام العينة و d. استنادًا إلى وظيفة الارتباط الذاتي، تُظهر النتائج أنه الارتباط يتناقص ببطء شديد، مما يعني أن العملية بها ذاكرة طويلة.

Introduction

The Long-memory or long-range dependence (LRD), property describes the high-order correlation structure of a series. If a series exhibits long memory, there is persistent temporal dependence even between distant observations. Fractionally integrated processes that are associated with hyperbolically decaying autocorrelations can give rise to long memory (Beran, 1994). The autoregressive fractionally integrated moving average ARFIMA (p, d, q) model was introduced by Granger and Joyeux (1980). Since then there has been great studies in the estimation of long- memory modelling e.g. Granger and Joyeux (1980), Hosking (1981), Geweke and Portter-Hudack (1983), Sowel (1992) and Mayoral (2007). The characteristics of ARFIMA (p,d,q) processes when $d \in (-0.5, 0.5)$ are, the process is stationary and invertible . For $d \in (0, 0.5)$, the process is long memory and its covariance is stationary while its variance is finite. Furthermore when $d \in (-0.5, 0)$, the process is identified as having intermediate memory, since autocorrelation is always negative (anti-persistent), while when d=0 the process is stationary with short memory.

Estimating Of Fractional Differencing Parameter (d)

There are many estimators of the parameter d proposed in the literature. They are grouped mainly into two categories: The semi-parametric and parametric methods. In the first group one finds, for example, Geweke and Porter-Hudak (1983), Reisen (1994), Chen, et al. (1994), Robinson (1995) and Lobato and Robinson (1996) and others. In the second category are Fox and Taqqu (1986), Dahlhaus (1989) and Sowell (1992). Some recent simulation studies comparing different techniques of estimation in long-memory process may be found in Taqqu et al. (1995), Bisaglia et al. (1998), Taqqu and Teverovsky (1996), Reisen and Lopes (1999) and Hurvich and Deo (1999). In this paper, we present only three estimator methods for d by using periodoagram, smoothed period-diagram and Whittle methods.

The methods are summariezed as follows:

Periodogram Estimator ($\widehat{d_p}$)

The regression of the periodogram proposed by Geweke and Porter-Hudak (GPH)(1983), denoted by ($\widehat{d_p}$), used the periodogram function $I(w)$. The number of observations in the regression equation is a function g(n) of the sample size n where g(n) = n^{α}, $0 < \alpha < 1$. Geweke and Porter-Hudak (1983) showed that ($\widehat{d_p}$) is asymptotically normally distributed with E($\widehat{d_p}$) = d and $var\,(\widehat{d_p}) = \frac{\pi^2}{6\sum_{i=1}^{g(n)}(x_i - \bar{x})^2}$, where

$x_i = ln\,(2\sin(w_j/2))^2$.

Smoothed Periodogram Estimator ($\widehat{d_{sp}}$)

The regression estimator using the smoothed periodogram function suggested by Reisen (1994), denoted by ($\widehat{d_{sp}}$). This regression estimator is obtained by replacing the spectral density function by the smoothed periodogram function with the Parzen lag window. In this method, g(n) is chosen as above and the truncation point in the Parzen lag window is

$$m = n^{\beta},\ 0 < \beta < 1.$$

Reisen (1994) showed that ($\widehat{d_{sp}}$) is asymptotically normally distributed with E($\widehat{d_{sp}}$)= d and var($\widehat{d_{sp}}$) $\approx 0.539285\,\frac{m}{n\sum_{i=1}^{g(n)}(x_i - \bar{x})^2}$.

Whittle Estimator ($\widehat{d_w}$)

The parametric method considered, hereafter, denoted by ($\widehat{d_w}$), was proposed by Fox and Taqqu (1986), by adapting the approach suggested by Whittle (1953). The estimator ($\widehat{d_w}$) is based on the periodogram and it involves the function

$$Q(\zeta) = \int_{-\pi}^{\pi} \frac{I(w)}{f(w,\ \zeta)}\, dw$$

Where, $(w,, \zeta)$ is the spectral density at frequency ▯ and ζ denotes the vector that contains the parameter d and also all the unknown autoregressive and moving average parameters. The Whittle estimator is the value of ζ which minimizes the function Q(.). For computational purposes the estimator ($\widehat{d_w}$) is obtained by using the discreet form of Q(.), as in Dahlhaus (1989, page 1753), that is,

$$Ln(\zeta) = \frac{1}{2n}\sum_{j=1}^{n-1} \{\, ln\, f(w_j,\ \zeta) + \frac{I(w_j)}{f(w_j, \zeta)} \}$$

For more detail see Fox and Taqqu (1986). The Whittle estimator is the value of ζ which minimizes the function.

Methodology

We have conducted simulation studies to obtain some information about the degree of differencing parameter from ARFIMA (0, d^*, 0) where $d^* \in (0.0, 0.5)$. In this study we generated a time series with parameter d^* = 0.1, 0.3 and 0.45, and sample sizes n = $\mathbf{2}^{\boldsymbol{m}}$, where $\boldsymbol{m}$ = 6, 7, 8, 9, 10, 11. Next, we estimated d using three methods. Two of them are semi-parametric. These are the $\widehat{\boldsymbol{d}_{sp}}$, $\widehat{\boldsymbol{d}_{sp}}$) and the $\widehat{\boldsymbol{d}_{w}}$. For each kind of time series, we repeatedly carried out the procedure 1000 times. Accordingly, we reported the average values of the estimates of $\widehat{\boldsymbol{d}}$, the corresponding sample standard deviations, the bias and the mean square errors.

The following notation is used. If $d^*$is the nominal value of d and $\boldsymbol{d_i}$ is the estimate for sample $\boldsymbol{i}$ then,

$$\overline{\boldsymbol{d}} = \frac{1}{1000}\sum_{i=1}^{1000} \boldsymbol{d_i}\,,\ \ \widehat{\boldsymbol{\sigma}}^2 = \frac{1}{999}\sum_{i=1}^{1000}(\boldsymbol{d_i} - \overline{\boldsymbol{d}})^2 \text{ and } \ \text{MSE} = \frac{1}{1000}\sum_{i=1}^{1000}(\boldsymbol{d_i} - \boldsymbol{d}^*)^2.$$

The results obtained considering β = 0.9 in the truncation point for the smoothed periodogram function in the $\widehat{\boldsymbol{d}_{sp}}$ estimator. For $\widehat{\boldsymbol{d}_{p}}$ and $\widehat{\boldsymbol{d}_{sp}}$ estimators we consider g(n)= $\boldsymbol{n}^{\boldsymbol{\alpha}}$= $\boldsymbol{n}^{\mathbf{0.5}}$ and the R 3.0.0 software package were used for data analysis.

Simulation Study

In this simulation study we followed the research methodology in order to obtain empirical results about estimating fractional differencing parameter in ARFIMA (0,d,0). The summary of results is as follows:

In Tables 1 and 2, we consider the estimation of $\boldsymbol{d}^* \in$ {0.1, 0.3, 0.45}. The best values of the bias (smallest absolute value), and smallest values of the standard deviation and the mean square error are presented in boldface. From Table 1, we can see that, for the case when $\boldsymbol{d}^*$ $\in$ {0.1, 0.3, 0.45}, with n = 64 and 128, also which is similar to $\boldsymbol{d}^*$= 0.3 with n = 256. In addition, $\widehat{\boldsymbol{d}_{p}}$ estimator presents good results in the sense of minimizing the bias. The mean values of $\widehat{\boldsymbol{d}_{sp}}$ and $\widehat{\boldsymbol{d}_{w}}$ underestimate the true parameter. It should be noted that n = 256 may not be large enough for some of the methods to perform better for n = 512, 1024, 2048 and 4096. The results indicate that the $\widehat{\boldsymbol{d}_{p}}$ and $\widehat{\boldsymbol{d}_{w}}$ estimators perform reasonably well and are very competitive and underestimate $\boldsymbol{d}^*$. The bias of all methods decrease substantially as n increases with $\widehat{\boldsymbol{d}_{w}}$ having the downward bias in the whole range of $\boldsymbol{d}^*$.

From the results summarized in Table 2, it is clear that all estimators present good results, in the sense of minimizing the standard deviation and the mean squared error values. However, the standard deviation and the mean square error of $\widehat{\boldsymbol{d}}$ based on $\widehat{\boldsymbol{d}_{p}}$ tends to be larger than that of other methods. The standard deviation and the mean squared error of the estimator calculated by $\widehat{\boldsymbol{d}_{w}}$ are also smaller than those of the estimator calculated by $\widehat{\boldsymbol{d}_{p}}$ and $\widehat{\boldsymbol{d}_{sp}}$. We can see similar a phenomenon at all sample sizes in the whole range of $\boldsymbol{d}^*$, where $\widehat{\boldsymbol{d}_{w}}$ is the best estimator compared with $\widehat{\boldsymbol{d}_{p}}$ and $\widehat{\boldsymbol{d}_{sp}}$. Although the $\widehat{\boldsymbol{d}_{sp}}$ estimator has better performance than $\widehat{\boldsymbol{d}_{p}}$ in terms of small standard deviation and mean squared error values, which is expected, since $\widehat{\boldsymbol{d}_{sp}}$ uses the smoothed periodogram function to estimate the spectral density function.

Table 1. Mean and bias of parameter estimation $\widehat{\boldsymbol{d}}$ from ARFIMA (0, $\boldsymbol{d}^*$, 0) model.

n	Estimator	$\boldsymbol{d}^* = 0.1$		$\boldsymbol{d}^* = 0.3$		$\boldsymbol{d}^* = 0.45$	
		Mean	Bias	Mean	Bias	Mean	bias
64	$\widehat{\boldsymbol{d}_p}$	0.09810	-0.00190	0.30670	0.00670	0.46840	0.01840
	$\widehat{\boldsymbol{d}_{sp}}$	0.01395	-0.08605	0.20105	-0.09895	0.35038	-0.09962
	$\widehat{\boldsymbol{d}_w}$	0.08404	-0.01596	0.28624	-0.01376	0.41731	-0.03269
128	$\widehat{\boldsymbol{d}_p}$	0.08967	-0.01033	0.29334	-0.00666	0.45223	0.00223
	$\widehat{\boldsymbol{d}_{sp}}$	0.02781	-0.07219	0.21715	-0.08285	0.36828	-0.08172
	$\widehat{\boldsymbol{d}_w}$	0.08888	-0.01112	0.29116	-0.00884	0.43364	-0.01636
256	$\widehat{\boldsymbol{d}_p}$	0.09353	-0.00647	0.29838	-0.00162	0.46232	0.01232
	$\widehat{\boldsymbol{d}_{sp}}$	0.04785	-0.05215	0.24414	-0.05586	0.40053	-0.04947
	$\widehat{\boldsymbol{d}_w}$	0.09502	-0.00498	0.29759	-0.00241	0.44709	-0.00291
512	$\widehat{\boldsymbol{d}_p}$	0.10350	0.00350	0.30985	0.00985	0.46765	0.01765
	$\widehat{\boldsymbol{d}_{sp}}$	0.06591	-0.03409	0.26469	-0.03531	0.42028	-0.02972
	$\widehat{\boldsymbol{d}_w}$	0.09764	-0.00236	0.29936	-0.00064	0.45075	0.00075
1024	$\widehat{\boldsymbol{d}_p}$	0.10213	0.00213	0.30570	0.00570	0.46358	0.01358
	$\widehat{\boldsymbol{d}_{sp}}$	0.07465	-0.02535	0.27523	-0.02477	0.43182	-0.01818
	$\widehat{\boldsymbol{d}_w}$	0.09806	-0.00194	0.29922	-0.00029	0.45109	0.00109
2048	$\widehat{\boldsymbol{d}_p}$	0.10490	0.00490	0.30804	0.00884	0.46228	0.01228
	$\widehat{\boldsymbol{d}_{sp}}$	0.08400	-0.01600	0.28354	-0.01646	0.43945	-0.01055
	$\widehat{\boldsymbol{d}_w}$	0.09898	-0.00102	0.29971	-0.00029	0.45100	0.00100

Table 2. Standard deviation and Mean square error of parameer estimation $\widehat{\boldsymbol{d}}$ from ARFIMA(0, $\boldsymbol{d}^*$, 0) model.

n	Estimator	$\boldsymbol{d}^* = 0.1$		$\boldsymbol{d}^* = 0.3$		$\boldsymbol{d}^* = 0.45$	
		St Dev	MSE	St Dev	MSE	St Dev	MSE
64	$\widehat{\boldsymbol{d}_p}$	0.34160	0.01080	0.33370	0.01060	0.32790	0.01040
	$\widehat{\boldsymbol{d}_{sp}}$	0.23752	0.00751	0.24293	0.00768	0.24649	0.00779
	$\widehat{\boldsymbol{d}_w}$	0.12696	0.00401	0.12425	0.00393	0.09694	0.00307
128	$\widehat{\boldsymbol{d}_p}$	0.26844	0.00849	0.27375	0.00866	0.27248	0.00862
	$\widehat{\boldsymbol{d}_{sp}}$	0.19586	0.00619	0.20418	0.00646	0.20708	0.00655
	$\widehat{\boldsymbol{d}_w}$	0.08149	0.00258	0.08293	0.00262	0.06835	0.00216
256	$\widehat{\boldsymbol{d}_p}$	0.20860	0.00660	0.21674	0.00685	0.20564	0.00650
	$\widehat{\boldsymbol{d}_{sp}}$	0.15798	0.00500	0.16428	0.00520	0.16839	0.00532
	$\widehat{\boldsymbol{d}_w}$	0.05315	0.00168	0.05373	0.00170	0.04730	0.00150
512	$\widehat{\boldsymbol{d}_p}$	0.17212	0.00544	0.16895	0.00534	0.17186	0.00543
	$\widehat{\boldsymbol{d}_{sp}}$	0.13264	0.00419	0.13673	0.00432	0.14070	0.00445
	$\widehat{\boldsymbol{d}_w}$	0.03709	0.00117	0.03707	0.00117	0.03444	0.00109
1024	$\widehat{\boldsymbol{d}_p}$	0.13284	0.00420	0.13462	0.00426	0.13173	0.00417
	$\widehat{\boldsymbol{d}_{sp}}$	0.10561	0.00334	0.10868	0.00344	0.11080	0.00350
	$\widehat{\boldsymbol{d}_w}$	0.02517	0.00080	0.02536	0.00080	0.02515	0.00080
2048	$\widehat{\boldsymbol{d}_p}$	0.10627	0.00336	0.10617	0.00336	0.10975	0.00347
	$\widehat{\boldsymbol{d}_{sp}}$	0.08614	0.00272	0.08831	0.00279	0.09039	0.00286
	$\widehat{\boldsymbol{d}_w}$	0.01816	0.00057	0.01826	0.00058	0.01849	0.00059

Table 3 shows the confidence interval and testing of hypothesis for $\boldsymbol{d}^{*}= 0.1$. From the results, we can see that the $\widehat{\boldsymbol{d_p}}$ estimator is acceptable for all sample sizes, while the $\widehat{\boldsymbol{d_w}}$ is

Table 3. Confidence interval and testing of hypothesis: H_0: $\boldsymbol{d}^{*}= 0.1$ vs H_1: $\boldsymbol{d}^{*}\neq 0.1$.

n	Estimator	Mean	95% CI	T	P-value	Decision
64	$\widehat{\boldsymbol{d_p}}$	0.0981	(0.0769; 0.1192)	-0.18	0.857	Accept
	$\widehat{\boldsymbol{d_{sp}}}$	0.0140	(-0.00079; 0.02869)	-11.46	0.000	Reject
	$\widehat{\boldsymbol{d_w}}$	0.0840	(0.07616; 0.09192)	-3.98	0.000	Reject
128	$\widehat{\boldsymbol{d_p}}$	0.0897	(0.07301; 0.10633)	-1.22	0.224	Accept
	$\widehat{\boldsymbol{d_{sp}}}$	0.0278	(0.01565; 0.03996)	-11.66	0.000	Reject
	$\widehat{\boldsymbol{d_w}}$	0.0889	(0.08383; 0.09394)	-4.31	0.000	Reject
256	$\widehat{\boldsymbol{d_p}}$	0.0935	(0.08058; 0.10647)	-0.98	0.327	Accept
	$\widehat{\boldsymbol{d_{sp}}}$	0.0479	(0.03805; 0.05765)	-10.44	0.000	Reject
	$\widehat{\boldsymbol{d_w}}$	0.0950	(0.09172; 0.09832)	-2.96	0.003	Reject
512	$\widehat{\boldsymbol{d_p}}$	0.1035	(0.09282; 0.11418)	0.64	0.521	Accept
	$\widehat{\boldsymbol{d_{sp}}}$	0.0659	(0.05768; 0.07414)	-8.13	0.000	Reject
	$\widehat{\boldsymbol{d_w}}$	0.0976	(0.09534; 0.09994)	-2.01	0.045	Reject
1024	$\widehat{\boldsymbol{d_p}}$	0.1021	(0.09389; 0.11038)	0.51	0.612	Accept
	$\widehat{\boldsymbol{d_{sp}}}$	0.0747	(0.06810; 0.08120)	-7.59	0.000	Reject
	$\widehat{\boldsymbol{d_w}}$	0.0981	(0.09649; 0.09962)	-2.44	0.015	Reject
2048	$\widehat{\boldsymbol{d_p}}$	0.1049	(0.09830; 0.11149)	1.46	0.145	Accept
	$\widehat{\boldsymbol{d_{sp}}}$	0.0840	(0.07866; 0.08935)	-5.87	0.000	Reject
	$\widehat{\boldsymbol{d_w}}$	0.0990	(0.09785; 0.10010)	-1.78	0.075	Accept

acceptable only when the sample size very large at n = 2048.

In Table 4, the results of confidence interval and testing of hypothesis for $\boldsymbol{d}^{*}= 0.3$, show that the $\widehat{\boldsymbol{d_p}}$ estimator has the same results shown in the Table 3, except when the sample size n = 2048. While the $\widehat{\boldsymbol{d_w}}$ estimator has better results compared with $\boldsymbol{d}^{*}= 0.1$ in Table (3). The results of confidence interval and testing of hypothesis for $d^{*}= 0.45$, in Table 5 showed that the performance of $\widehat{\boldsymbol{d_w}}$ estimator is more powerful for the sample size n $\geqq$ 256, comparing with the performance of $\widehat{\boldsymbol{d_p}}$ estimator.

In general, as n increases the estimators get even better. Except for the $\widehat{\boldsymbol{d_{sp}}}$ estimator, the other methods tends to estimate the true parameter. Furthermore, the $\widehat{\boldsymbol{d_w}}$ estimator seems to be more accurate; smaller bias, SD and MSE, than the other estimators. Testing of hypothesis results showed that, the estimates of the fractional differencing parameter d by Whittle's method showed good performance with increase of sample sizes and d. So that we depended on it to estimate the true parameter for real time series data which is under research and study.

Table 4. Confidence interval and testing of hypothesis: H_0: $\boldsymbol{d}^{*} = 0.3$ vs H_1: $\boldsymbol{d}^{*} \neq 0.3$.

N	Estimator	Mean	95% CI	T	P-value	Decision
64	$\widehat{d_p}$	0.3067	(0.28600; 0.32740)	0.64	0.523	Accept
	$\widehat{d_{sp}}$	0.20105	(0.18597; 0.21612)	-12.88	0.000	Reject
	$\widehat{d_w}$	0.28624	(0.27853; 0.29395)	-3.50	0.000	Reject
128	$\widehat{d_p}$	0.29334	(0.27636; 0.31033)	-0.77	0.442	Accept
	$\widehat{d_{sp}}$	0.21715	(0.20448; 0.22983)	-12.83	0.000	Reject
	$\widehat{d_w}$	0.29116	(0.28601; 0.29630)	-3.37	0.001	Reject
256	$\widehat{d_p}$	0.29838	(0.28493; 0.31182)	-0.24	0.813	Accept
	$\widehat{d_{sp}}$	0.24414	(0.23394; 0.25433)	-10.75	0.000	Reject
	$\widehat{d_w}$	0.29759	(0.29426; 0.30093)	-1.42	0.157	Accept
512	$\widehat{d_p}$	0.30985	(0.29936; 0.32033)	1.84	0.066	Accept
	$\widehat{d_{sp}}$	0.26469	(0.25620; 0.27317)	-8.17	0.000	Reject
	$\widehat{d_w}$	0.29936	(0.29706; 0.30166)	-0.55	0.585	Accept
1024	$\widehat{d_p}$	0.30570	(0.29735; 0.31406)	1.34	0.181	Accept
	$\widehat{d_{sp}}$	0.27523	(0.26849; 0.28198)	-7.21	0.000	Reject
	$\widehat{d_w}$	0.29922	(0.29765; 0.30079)	-0.97	0.330	Accept
2048	$\widehat{d_p}$	0.30804	(0.30145; 0.31463)	2.39	0.017	Reject
	$\widehat{d_{sp}}$	0.28354	(0.27806; 0.28902)	-5.89	0.000	Reject
	$\widehat{d_w}$	0.29971	(0.29857; 0.30084)	-0.51	0.609	Accept

Autocorrelation for Long Memory

The detection of long-range dependence in time series analysis is an important task, as we know the theoretical definition of a long-memory (or long-range dependent) process is based on the autocorrelation function, where the autocorrelation declines hyperbolically to zero when the lag length increases.

The model ARFIMA(0,d,0) can be investigated further by analysing the behavior of the autocorrelation function. The results are displayed with different values of d and sample size n. Figures (1-3) provide three graphs of three models of ARFIMA(0, 0.1, 0), ARFIMA(0, 0.3, 0) and ARFIMA(0,0.45,0), with sample sizes of 64, 28 and 256. In each Figure, the series on the three graphs show indistinguishable differences between them without proper statistical tools. This can solve this problem using autocorrelation as a tool designed for the properties of long memory with autocorrelation.

Table 5. Confidence interval and testing of hypothesis: H_0: $\boldsymbol{d}^*= 0.45$ vs H_1: $\boldsymbol{d}^*\neq 0.45$.

N	Estimator	Mean	95% CI	T	P-value	Decision
64	$\widehat{d_p}$	0.4684	(0.44800; 0.48870)	1.77	0.077	Accept
	$\widehat{d_{sp}}$	0.3504	(0.33509; 0.36568)	-12.78	0.000	Reject
	$\widehat{d_w}$	0.4173	(0.41130; 0.42333)	-10.66	0.000	Reject
128	$\widehat{d_p}$	0.4522	(0.41130; 0.42333)	0.26	0.796	Accept
	$\widehat{d_{sp}}$	0.3683	(0.35543; 0.38113)	-12.48	0.000	Reject
	$\widehat{d_w}$	0.4336	(0.42939; 0.43788)	-7.57	0.000	Reject
256	$\widehat{d_p}$	0.4623	(0.44956; 0.47508)	1.89	0.059	Accept
	$\widehat{d_{sp}}$	0.4005	(0.39008; 0.41098)	-9.29	0.000	Reject
	$\widehat{d_w}$	0.4471	(0.44416; 0.45003)	-1.94	0.052	Accept
512	$\widehat{d_p}$	0.4677	(0.45698; 0.47831)	3.25	0.001	Reject
	$\widehat{d_{sp}}$	0.4203	(0.41155; 0.42901)	-6.68	0.000	Reject
	$\widehat{d_w}$	0.4508	(0.44861; 0.45289)	0.69	0.491	Accept
1024	$\widehat{d_p}$	0.4636	(0.45540; 0.47175)	3.26	0.001	Reject
	$\widehat{d_{sp}}$	0.4318	(0.42495; 0.43870)	-5.19	0.000	Reject
	$\widehat{d_w}$	0.4511	(0.44953; 0.45265)	1.37	0.172	Accept
2048	$\widehat{d_p}$	0.4623	(0.45547; 0.46909)	3.54	0.000	Reject
	$\widehat{d_{sp}}$	0.4395	(0.43385; 0.44506)	-3.69	0.000	Reject
	$\widehat{d_w}$	0.4510	(0.45005; 0.45234)	1.66	0.051	Accept

Figures 4-6, provide three graphs for the ACF of the three ARFIMA models, with sample sizes 64, 128 and 256. It is observed here, that the correlation decays very slowly. The intuitive interpretation is that the process has a long memory. Although in the case of ARFIMA(0,0.1,0) process, the autocorrelations decays to zero so fast. In this case, obviously, the process is not affected by the long memory, because the process with a small value of the parameter closes to zero. That is why, there is a class of short memory that is easily confused with long memory processes (misspecification).

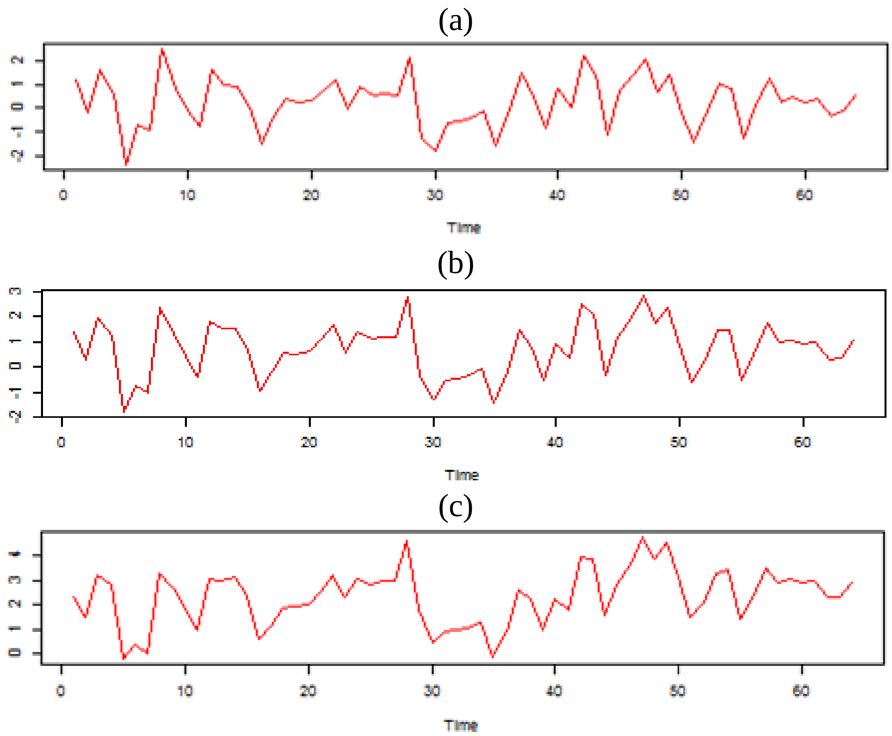
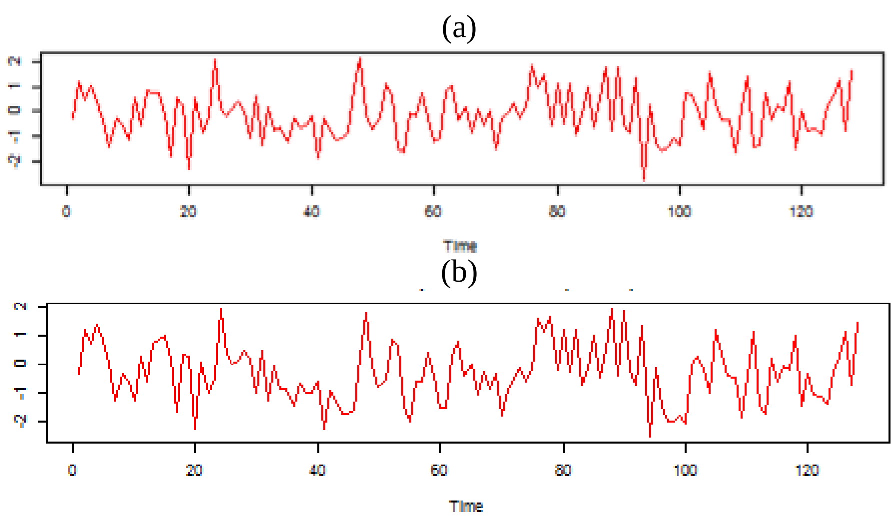


Fig. 1. Simulated time series for ARFIMA(0, $\boldsymbol{d}^*$,0) processes with fixed sample size 64 and different values of $\boldsymbol{d}^*$(a) $\boldsymbol{d}^*$= 0.1, (b) $\boldsymbol{d}^*$= 0.3 and (c) $\boldsymbol{d}^*$= 0.45.



(c)

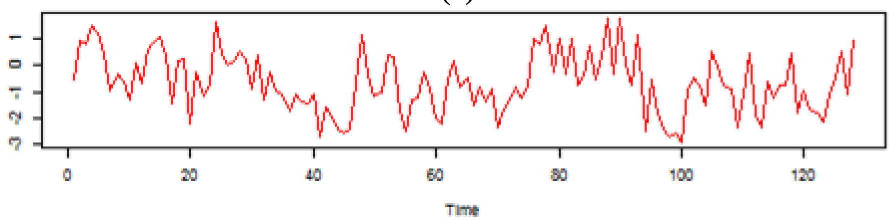


Fig. 2. Simulated time series for ARFIMA(0, $\boldsymbol{d}^*$,0) processes with fixed sample size 128 and different values of $\boldsymbol{d}^*$(a) $\boldsymbol{d}^*$= 0.1, (b) $\boldsymbol{d}^*$= 0.3 and (c) $\boldsymbol{d}^*$= 0.45.

(a)

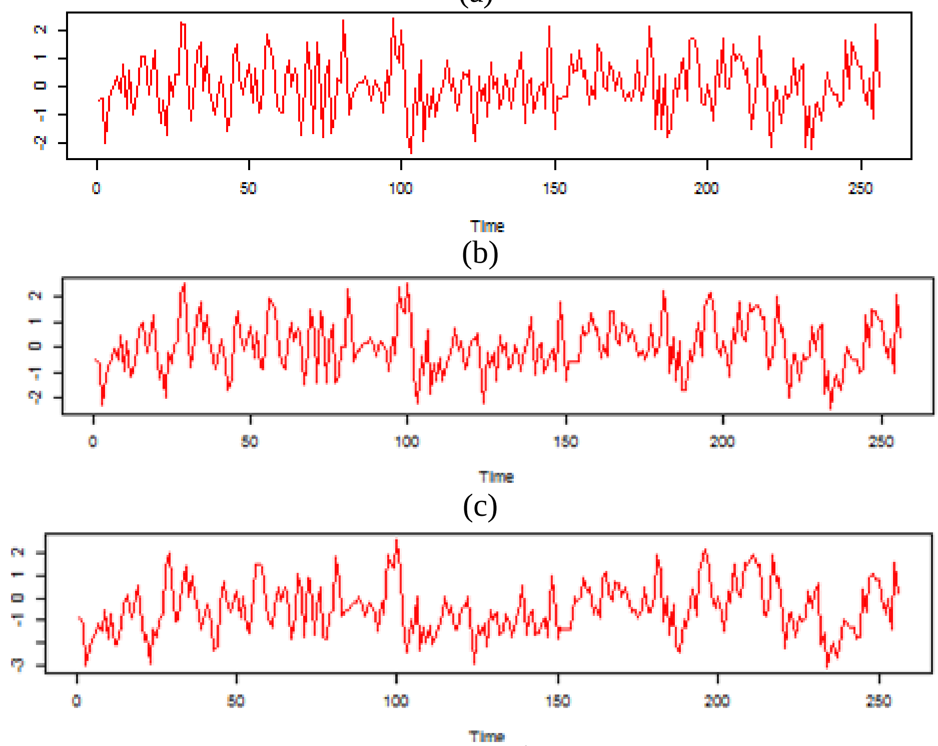
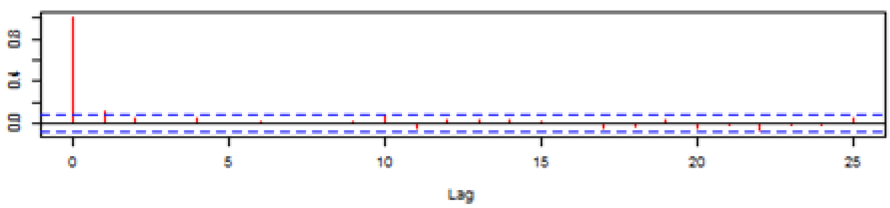
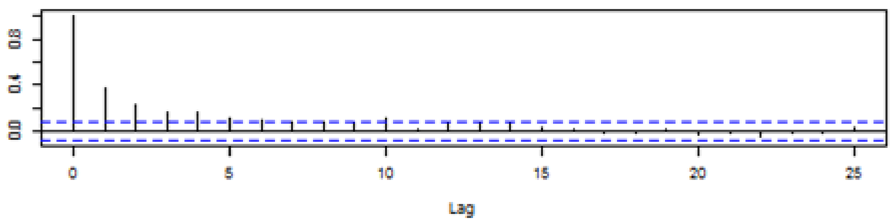


Fig. 3. Simulated time series for ARFIMA(0, $\boldsymbol{d}^*$,0) processes with fixed sample size 256 and different values of $\boldsymbol{d}^*$(a) $\boldsymbol{d}^*$= 0.1, (b) $\boldsymbol{d}^*$= 0.3 and (c) $\boldsymbol{d}^*$= 0.45.

(a)



(b)



(c)

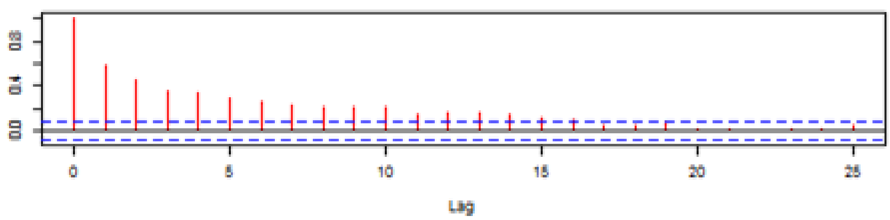
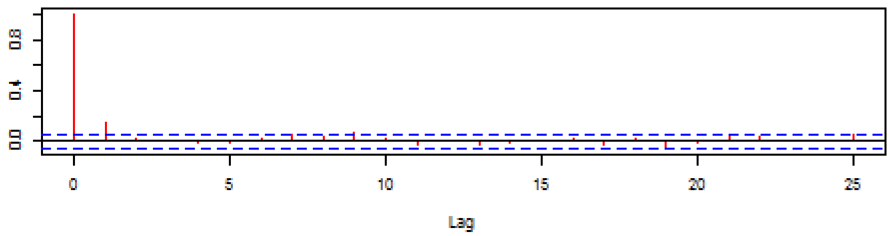


Fig. 4. ACF of ARFIMA(0,$\boldsymbol{d}^*$,0) processes with sample size 64, and different values of $\boldsymbol{d}^*$(a) $\boldsymbol{d}^*$= 0.1, (b) $\boldsymbol{d}^*$= 0.3 and (c) $\boldsymbol{d}^*$= 0.45.

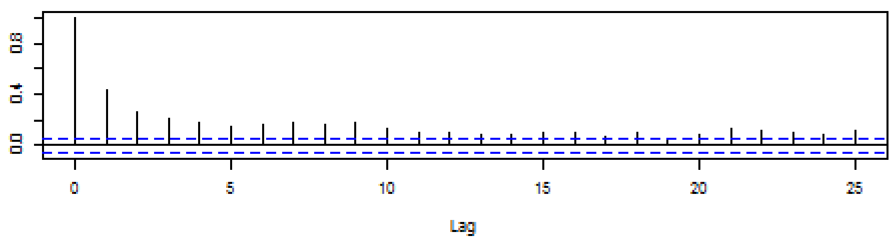
Conclusion

In this work, we have studied how to fit autoregressive fractionally integrated moving average ARFIMA models, as a solution towards the non-stationary persistency of time series in the long run data. Hence, we analyzed the estimation of the degree of differencing d in ARFIMA(0, d, 0) process, when it belongs to the interval (0, 0.5). We present a simulation study for the estimators of d, among the methods of estimating the parameter of the ARFIMA model. Three methods were used; Geweke-Proter- Hudak's (GPH) estimator ($\widehat{\mathbf{d_p}}$), smoothed periodogram ($\widehat{\mathbf{d_{sp}}}$) estimator and Whittle's estimator ($\widehat{\mathbf{d_w}}$). We compared them by simulation based on artificially generated time series by ARFIMA(0, d, 0) with d $\in$ {0.1, 0.3, 0.45} and different sample sizes (64, 128, 256, 512, 1024, 2048) and 1000 repetitions for each sample.

(a)



(b)



(c)

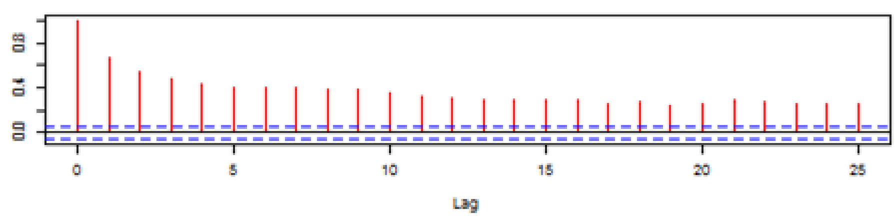
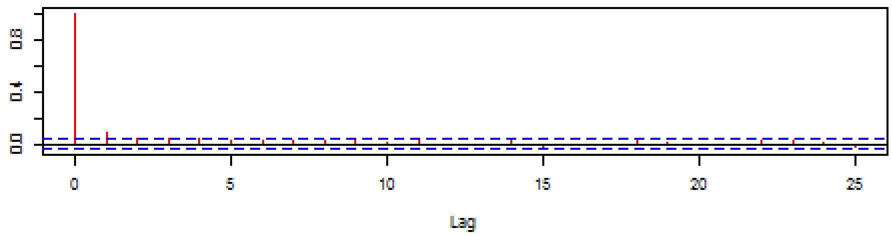


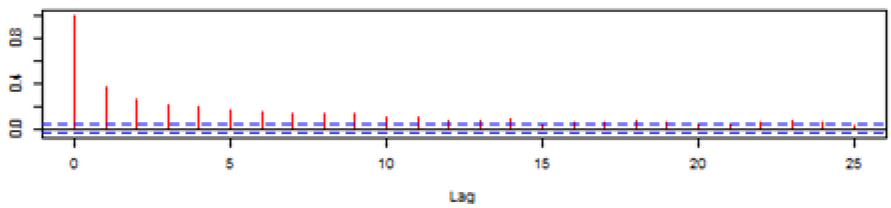
Fig. 5. ACF of ARFIMA(0,$\boldsymbol{d}^*$,0) processes with sample size 128 and different values of $\boldsymbol{d}^*$(a) $\boldsymbol{d}^*$= 0.1, (b) $\boldsymbol{d}^*$= 0.3 and (c) $\boldsymbol{d}^*$= 0.45.

The results indicate that the performance of the $\widehat{\mathbf{d_w}}$ estimator is usually good compared with the other semi-parametric methods $\widehat{\mathbf{d_p}}$ and $\widehat{\mathbf{d_{sp}}}$; it has small standard deviation and mean squared error for all cases. Also as n increases more than 256, the average of $\hat{\mathbf{d}}$ was nearer to the true value with smaller bias than the other estimators were. The estimator based on smoothed periodogram has significant downward bias, but has better performance than $\widehat{\mathbf{d_p}}$ in terms of small standard deviation and mean squared error. The $\widehat{\mathbf{d_p}}$ estimator presents good results in the sense of minimizing the bias when the sample size is less than 256. The testing of hypothesis results showed that, the estimates of the fractional differencing parameter d by $\widehat{\mathbf{d_w}}$ has good performance with increase of sample sizes and d. So that we depended on it to estimate the true parameter for real time series data which is under study. On other hand, based on the behavior of autocorrelation function, the autocorrelation declines hyperbolically to zero when the lag length increases. The results displayed with different values of d and sample size n, based on three models ARFIMA(0, d, 0) d= 0.1 , 0.3, 0.45. It was observed that the correlation decays very slowly. The intuitive interpretation is

(a)



(b)



(c)

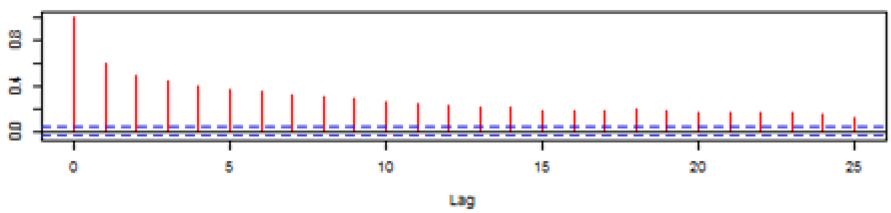


Fig. 6. ACF of ARFIMA(0,$\mathbf{d}^*$,0) processes with sample size 256 and different values of $\boldsymbol{d}^*$(a) $\boldsymbol{d}^*$= 0.1, (b) $\boldsymbol{d}^*$= 0.3 and (c) $\boldsymbol{d}^*$= 0.45.

that the process has a long memory. Although in the case of ARFIMA(0,0.1,0) process the autocorrelations decay to zero was so fast. In this case, obviously the process is not affected by long memory, because the process with a small value of the parameter closes to zero.

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