The Libyan Journal of Science
University of Tripoli Vol. 26, No. 02
https://uot.ed.ly/journals/index.php/ljs

Half Life of Alpha Decay from A Straight Line Potential Barrier

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ARTICLE INFO

Article history:

Received 01/10/2023

Received in revised form 29/11/2023

Accepted 05/12/2023

ABSTRACT

Following Gamow theory of alpha decay, we assumed that the nuclear interior potential well can be manifested by a new potential well on the form of straight line equation. In this work, the nuclear potential well brought from rigidity to a rather flexible nature, where the slope of straight line can be varied to maintain optimum outputs. The results of our calculations showed good agreement with experiment.

Keywords: Tunneling, Coulomb potential, potential well, transmission coefficient, half life.

1. Introduction

The decay of alpha particle has been a source of debates for physicists for some time. According to classical theory, a positively charged alpha particle encounters a repulsive coulomb potential near the nucleus of an atom, $V(r) = \frac{zZ_De^2}{r}$, where ze is the charge of alpha particle and Z_De is charge of the daughter nucleus, with the affect of repulsion tends to decrease abruptly at nuclear radius R. The energy necessary for an alpha particle to escape from the nucleus is compared with the potential at its local maximum, $r = R$, which is high enough to assure no spontaneous alpha emission, however, alpha emissions occur frequently in nature, with energies several times smaller than the predicted energy minimum. It becomes obvious that classical theory inadequate to describe the physics behind nuclear decay. In addition, classical physics did not explain the wide range of half lives of decaying particles via alpha emission, which extends from nanoseconds to billions of years. On the other hand, the quantum mechanics offers an alternative description, accordingly, a particle partially bound within a finite potential well has a certain probability of being transmitted through the potential barrier. In the year 1928, Gamow and, Gurney and Condon, independently were able to provide a quite answer to the puzzle [1]. These authors assumed that the alpha particle confined within the nucleus by the stimulating effect of coulomb potential $V(r)$. There would be various approximations to determine the probability P that an alpha particle of energy E_α penetrates the potential barrier. The semi classical approach being the ratio of transmission coefficients [2], of the form (equation 1)

$$P \approx e^{-2\gamma} \quad \text{.........(1)}$$

where $\gamma = \sqrt{\frac{2m}{\hbar^2}}\int_{r_0}^{r} dr\;[V(r)-E_\alpha]^{\frac{1}{2}}$, accordingly, the decay constant of alpha particle per unit time is expressed unlike by authors [2], [3], [4], [5], we use (equation 2).

$$\lambda = \frac{v_i}{2R}P \quad \text{…………. (2)}$$

where v_i denotes the speed of alpha particle inside the nucleus, with the corresponding decay half life (equation 3):

$$t_{1/2} = \frac{\ln(2)}{\lambda} \quad \text{.. ……….(3)}$$

This paper is planned as following: In section 2, we give a brief review of quantum tunneling in connection with Gamow theory of alpha penetration. In section 3, we present our approach and formalism, followed by the result of our calculations. Finally, in section 4, we draw our conclusion and discuss future directions for this work.

2. Quantum tunneling and alpha penetration

One of the most astonishing effects described by quantum mechanics is the transmission of a beam of particles through a potential barrier whose height is quite greater than the particle energy, as shown in Figure 1.

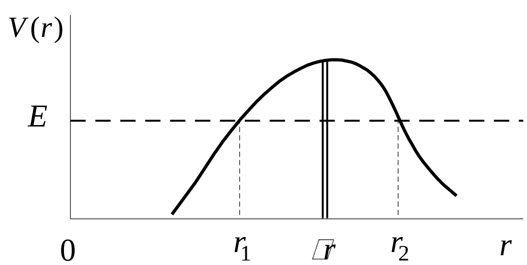

Fig. 1. Arbitrary shaped potential barrier.

By assuming that each particle of mass m has a total energy E, the attenuation of the corresponding wave function ψ across a tiny slice of interval δr of the potential barrier $V(r)$ can take the form of (equation 4).

$$\psi(r+\delta r) = \psi(r)e^{-\alpha(r)\delta r} \quad \text{……...(4)}$$

where $\alpha^2(r) = 2m[V(r)-E]/\hbar^2$. Approximating the left side and the right side exponential by the first two terms of their Taylor expansions (equation 5).

$$\psi(r) + \frac{d\psi(r)}{dr}\delta r \approx \psi(r)[1-\alpha(r)\delta r] \quad \text{……….(5)}$$

and integrating the results within the turning points r_1 and r_2, obtaining (equation 6).

$$\frac{\psi(r_2)}{\psi(r_1)} \approx e^{-\int_{r_1}^{r_2}\alpha(r)dr} \quad \text{………………..(6)}$$

this implies that the transmission probability P of particles across the potential barrier is (equation 7)

$$P = \frac{|\psi(r_2)|^2}{|\psi(r_1)|^2} \approx e^{-2\gamma} \quad \text{…………..…….(7)}$$

where $\gamma = \int_{r_1}^{r_2}\alpha(r)dr$. For the case of zero angular momentum, i.e., $l = 0$, the same mathematical expression holds for the probability $P \approx e^{-2\gamma}$ as well, with (equation 8)

$$\gamma = \sqrt{2m/\hbar^2}\int_{r_1}^{r_2}\sqrt{V(r)-E}\,dr \quad \text{…………(8)}$$

However, if the potential $V(r)$ is due to the mutual influence of two particles, as in the case of alpha decay, then the reduced mass of the products is involved of the form, $m = m_1 m_2/(m_1+m_2)$. The mechanism of alpha decay according to the theory of Gamow and, Gurney and Condon is that, the alpha particle exists inside the potential well, formed by nuclear and coulomb forces, as shown approximately in Figure 2, where the exact depth of the potential within the nucleus does not affect the final results.

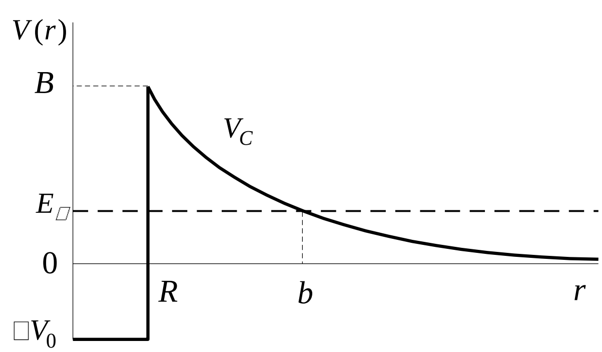

Fig.2. Nuclear potential well and the Coulomb potential V_C.

For a system consisting of an alpha particle and a daughter nucleus of charges $2e$ and $Z_D e$, and masses 4 and m_D, respectively, the corresponding potential energy in cgs units, is therefore

$$V(r) = \frac{2Z_D e^2}{r} \text{ ....(9)}$$

and the reduced mass of the system in amu:

$$m = \frac{4\, m_D}{4+m_D} \text{ ....(10)}$$

keeping the formulae general, the Gamow factor γ is now (equation11)

$$\gamma = \sqrt{\frac{2m}{\hbar^2}} \int_R^b \left(\frac{2Z_D e^2}{r} - E_\alpha\right)^{\frac{1}{2}} dr \text{..........(11)}$$

This integration can be evaluated straight forwards, but the transmission probability of alpha particle $e^{-2\gamma}$ very sensitive to a tiny variation in γ, in other words approximations are performed to assist the final result of computations.

3. Approach and Formalism

In this section, we suggest a modified potential barrier as an alternative to nuclear potential well, which is assumed on the form of a simple straight line equation with increasing positive slope σ, to align the straight line almost close to the vertical axis, as shown in Figure 3. The potential functions read as

$$V(r) = \begin{cases} \sigma r - \beta & a \le r \le R \\ \frac{2Z_D e^2}{r} & R \le r \le b \end{cases} \text{..........(12)}$$

where β denotes the intercept with the vertical axis, not shown in the Figure.

Next, is to evaluate γ in the limits $a \le r \le b$ as following:

$$\gamma = \sqrt{\frac{2m}{\hbar^2}} \int_a^b (V(r) - E_\alpha)^{\frac{1}{2}} dr \text{ ....(13)}$$

with the result of integration

$$\gamma = \frac{2}{3\sigma}\sqrt{\frac{2m}{\hbar^2}}\left(\frac{2Z_D e^2}{R} - E_\alpha\right)^{\frac{3}{2}} + b\sqrt{\frac{2mE_\alpha}{\hbar^2}}\left(\cos^{-1}\sqrt{\frac{R}{b}} - \sqrt{\frac{R}{b}\left(1-\frac{R}{b}\right)}\right) \text{ .....(14)}$$

where $b = 2Z_D e^2/E_\alpha$. This equation contains one free parameter σ, and will be used throughout our calculations.

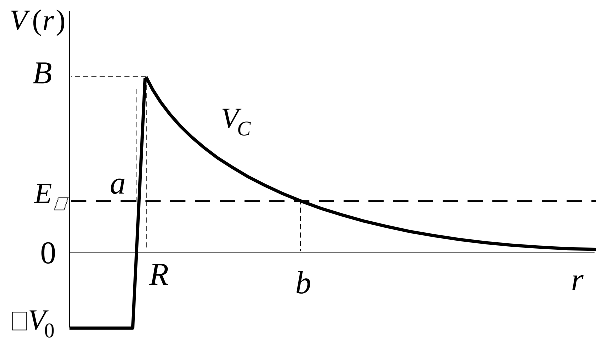

Fig.3. Modified nuclear potential well.

4. Results and discussion

The following calculations were based on cgs system of units:

$$e^2 = (4.8 \times 10^{-10} \text{ esu})^2 \text{ ....(15)}$$

$$m_\alpha = 4 \times 1.66 \times 10^{-24} \text{ g...........(16)}$$

$$\hbar^2 = (1.05 \times 10^{-27} \text{ erg s})^2 \text{............(17)}$$

We show in Table 1 a sample of the calculated half lives of alpha decay for different daughter nuclei compared with experiment. The last column presents the slope $\frac{V_c - E_\alpha}{R-a}$ obtained from fit to data in [1], [2]. The nuclear radius constant is taken as $r_0 = 1.4 \times 10^{-13}$ cm [6], where the effective distance is considered from the centers of the alpha-daughter nuclei [7] as

$$R = r_0 \left(4^{\frac{1}{3}} + A_D^{\frac{1}{3}}\right) \text{ cm ....(18)}$$

with the consideration of reduced mass of the alpha-daughter system

$$m = \left(\frac{4\, A_D}{4+A_D}\right) 1.66 \times 10^{-24} \text{ g..........(19)}$$

Noting that the whole computations including fits are straight forward, but are quite sensitive

Table1. A sample of computed half lives of alpha decay compared with experiment.

Daughter	A	Z	E_α [MeV]	Expt. $t_{1/2}$ [s]	Calc. $t_{1/2}$ [s]	$\sigma \times 10^8$ [erg/cm]
Th[1]	234	90	4.20	1.42×10^{17}	1.42×10^{17}	6.58
Ra[2]	228	88	4.05	4.39×10^{17}	4.39×10^{17}	4.39
Rn[2]	222	86	4.88	5.11×10^{10}	5.11×10^{10}	2.52
Ra[2]	224	88	5.52	6.00×10^{7}	6.00×10^{7}	2.62
Po[2]	218	84	5.59	3.31×10^{5}	3.31×10^{5}	2.41
Pb[2]	214	82	6.12	1.83×10^{2}	1.83×10^{2}	1.99
Pb[2]	212	82	6.89	1.60×10^{-1}	1.60×10^{-1}	1.90
Pb[2]	210	82	7.83	1.50×10^{-4}	1.50×10^{-4}	1.75
Pb[2]	208	82	8.95	3.00×10^{-7}	3.00×10^{-7}	1.38

5. Conclusion

In this work, the suggested potential well in the form of straight line equation to give enough flexibility to calculate decay parameters of alpha particle in a straight forward manner. The slope parameter σ actually deviates nuclear potential well from rigidity to a rather softer nature, which enables alpha particle to penetrate the potential barrier more realistic. The calculated half lives of a sample of nuclei, in the range from less than a micro second to billions of years showed good agreement with experiment. It is hoped to deduce an expression for the slope σ in such a way, and to attain full analytic results.

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